

too: there will always be plenty to keep us busy. So good-bye for now, my boy. First thing in the morning I'll be looking into the simplex algorithm for polytopic surfaces."

"The what?"

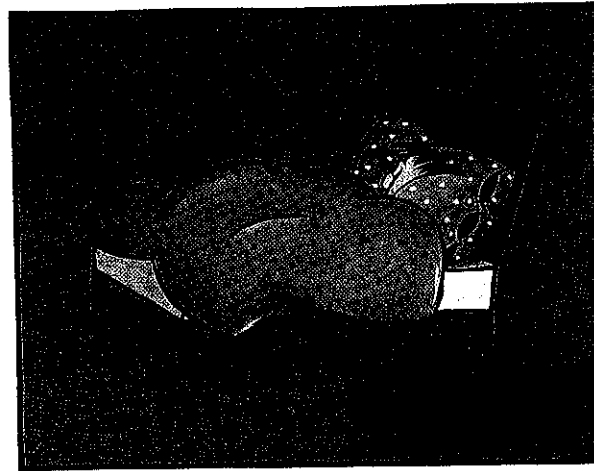
"The best way to unravel a foul-up. And for that I need a good night's sleep. I'm off to bed. Good night."

And with that, the number devil disappeared, leaving Robert to rock peacefully on the swinging bench.

I wonder what polytopic surfaces are, Robert thought, but then decided it didn't matter. All that mattered was that he didn't have to worry about Mr. Bockel anymore: whenever Mr. Bockel got on his back, the number devil would come to his rescue.

The night was mild and Robert gave himself up to the pleasure of rocking and dreaming, rocking and thinking about nothing at all till the break of day.

The Twelfth Night





Robert had stopped dreaming. There were no giant fish to gobble him up, no ants to crawl up his legs. Even Mr. Bockel and all his look-alikes left him in peace. He slid no more, he froze no more, he was no longer locked away in cellars. He slept better than he had ever slept before.

That was all well and good, but it also got to be rather boring. What was the number devil up to? Maybe he'd had a good idea and couldn't prove it. Or maybe he'd got bogged down in his polyp surfaces (or whatever it was he'd talked about last time).

Though maybe he'd forgotten about Robert.

Robert didn't like that idea in the least.

His mother couldn't understand why he spent hours on end drawing dots and lines and mumbling about visits to nonexistent friends in cities he'd never seen.

“Do your homework, Robert,” she would say. Once Mr. Bockel caught him scribbling away in class.

“What are you up to there, Robert? Give it here.”

But Robert had managed to crumple the paper with the huge number triangle into a ball and to sneak it over to Charlie. He could count on good old Charlie: Charlie was used to covering for him.

One night, however, he slept so soundly that it took him a long time to realize there was someone pounding on his door.

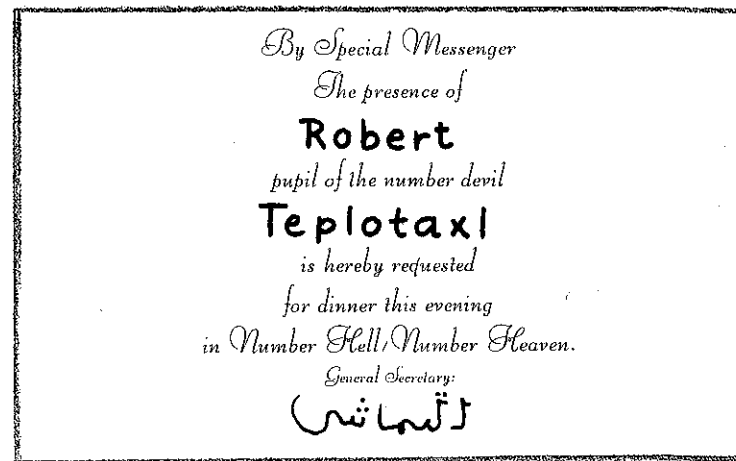
“Robert! Robert!”

And who did he see when he finally jumped out of bed and opened it but the number devil.

“I’m so glad to see you,” he said. “I’ve missed you.”



“Come quickly!” the number devil said. “I’ve got an invitation for you! Here!” And he handed him a printed card with a gold border. It said:



The signature was an illegible squiggle that appeared to be in Arabic script.

“So your name is Teplotaxl,” Robert said, pulling on his clothes as fast as he could. “Why didn’t you tell me before?”

“Only the inner circle can know a number devil’s name,” he replied.

“Does that mean I now belong to the inner circle?”

“Apparently. Otherwise you wouldn’t have received an invitation.”

“Funny,” Robert said. “The invitation says ‘Number Hell/Number Heaven.’ What’s that supposed to mean? Either one or the other?”

“Number Paradise, Number Hell, Number Heaven—it’s all the same in the end.” He went

over to the window and opened it as far as it would go. "You'll see. Ready?"

"Yes," said Robert, though he was starting to feel a bit uneasy about the whole thing.

"Then climb on my shoulders."

Robert was afraid he'd be too heavy for the number devil, who was no giant, but he didn't want to insult him. And, lo and behold, no sooner had he put his arms around the number devil's neck than the number devil leaped out of the window and soared into the air.

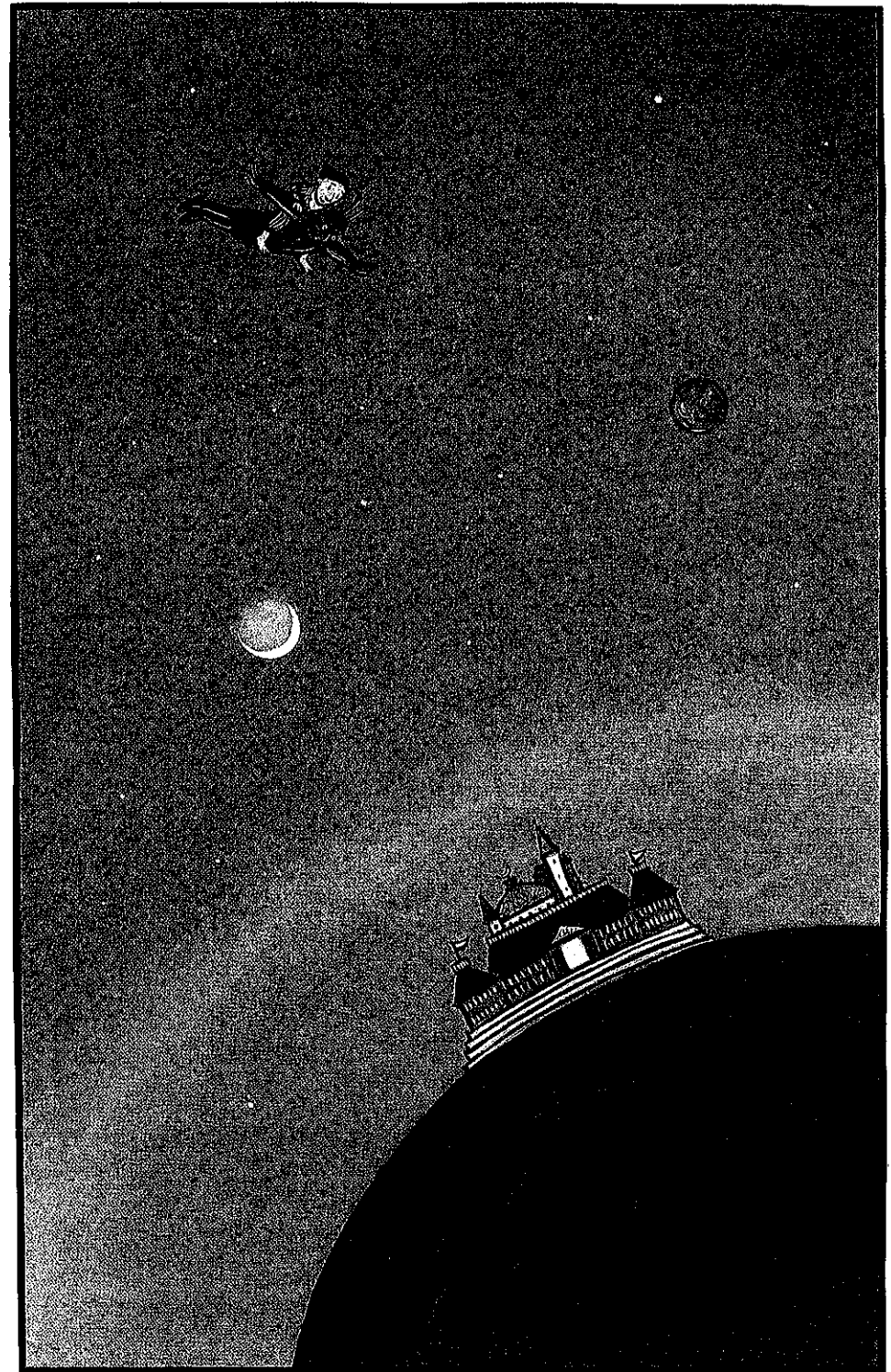
This could only happen in a dream, Robert thought. But why not? A flight without noise, without safety belts, without silly stewardesses giving you coloring books and plastic toys as if you were three years old—it was good for a change.

It ended in a perfect landing on the terrace of a magnificent palace.

"Here we are," said the number devil, letting Robert down.

"Where's my invitation?" asked Robert. "I'm afraid I left it at home."

"Don't worry," said the number devil. "Anyone who really wants to can enter. The trick is to get here in the first place. And *that*, as you can imagine, very few manage."



Robert had in fact noticed that nobody seemed to be checking the people going through the giant double door.

So in they went and set off down a long, long corridor with endless doors, many of which were open to varying degrees.

Robert peeked into the very first room. Teplotaxl put his finger to his lips and said, "Sh!" What they saw was a very old man with snow-white hair and a long nose. He was toddling round and round in circles and carrying out a great debate with himself.

"All Englishmen are liars," the man mumbled, "but if *I* say it, what then? I'm an Englishman myself. So I'm lying too. But then what I've just said—namely, that all Englishmen are liars—is not true. But if Englishmen tell the truth, then what I said before must be true as well. In other words, we *are* liars."

The number devil signaled to Robert, and on they went.

"That's poor Lord Rustle," Teplotaxl explained to his guest. "You remember, the one who proved $1 + 1 = 2$."

"Isn't he a bit... confused?" Robert asked. "Not that I'm surprised. He must be terribly old."

"He's not the least bit confused; he's got all his

wits about him. As for his age, age means nothing here. Besides, Lord Rustle is one of our youngest. He hasn't reached 150 yet."

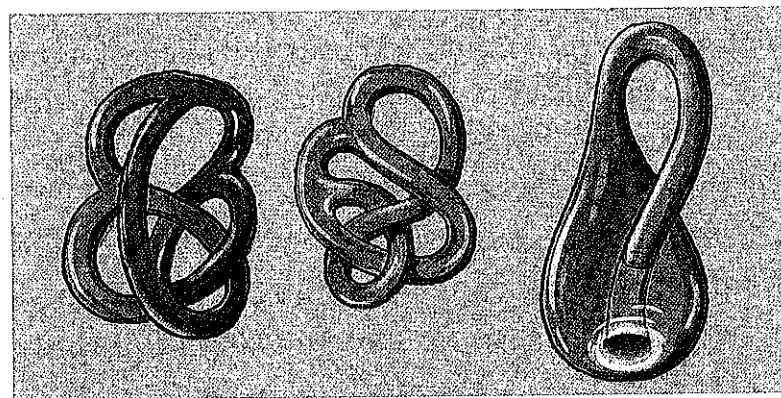
"So you have some pretty elderly residents here in the palace."

"You'll see," said Teplotaxl. "In Number Hell—I mean, Number Heaven—no one ever dies."

They came to another open door and peered in to see a man squatting in the corner. He was so tiny that Robert didn't notice him at first, though that may have been because the room was filled with the most curious objects. A few were large pretzels made of glass. Mr. Bockel would have been pleased, thought Robert, though you couldn't eat them and they had the weirdest shapes.

The number devil called Robert's attention to an unusual-looking green bottle.

"Examine it carefully," Teplotaxl whispered into



Robert's ear. "Can you tell the inside from the outside?"

Unbelievable! thought Robert. It could only exist in a dream.

"Imagine you wanted to paint the inside blue and the outside red. What would you do? There are no edges. You wouldn't know where to stop the blue or start the red."

"And that little man is the one who invented it?" asked Robert. "He looks as if he would be happier inside the bottle."

"Not so loud! Do you know what his name is? Dr. Little. Dr. Happy Little. But let's go now. We have lots more to see."

The next few doors they passed had DO NOT DISTURB signs on them, but one was wide open, and there they stopped. The walls and furniture were coated with a fine dust.

"That's no ordinary dust," said Teplotaxl. "It has more granules than a body could count in a lifetime. If you wanted to cover the head of a pin with it, you'd have to gather all the dust in this room. The man you see there is Professor Singer. He's the man who discovered the dust."

Professor Singer, a pale man with a goatee and piercing eyes, was singing to himself and doing a nervous little dance around the room. "Infinity

times infinity is infinity," he sang. "Superinfinity times infinity is superinfinity."

Let's get out of here, Robert thought.

His friend knocked politely on one of the next doors, and a friendly voice called out to them, "Come in!"

Teplotaxl was right: the palace's inhabitants were so old that he seemed a stripling by comparison. Nonetheless the two very old men who greeted them turned out to be quite lively.

"Welcome, gentlemen, welcome," said one of them, a man with large eyes and a wig. "My name is Owl, and this is Professor Horrors."

The latter looked very stern, and he scarcely glanced up from his papers. Robert had the feeling he was less than happy with their visit.

"We were just chatting about the prima-donna numbers," said the friendly one. "A fascinating topic, as I'm sure you are aware."

"Oh, yes," said Robert. "You never know where you stand with them."

"Right you are. But with the help of my colleagues I hope to get to the bottom of them."

"And what does Professor Horrors do, if I may ask?"

But Professor Horrors refused to divulge the subject of his labors.



“Professor Horrors is responsible for a brilliant discovery, a whole new category of number, in fact. Tell our friends here what you call them, will you?”

“Im,” said the man with the stern look about him. And that was all he said.

“He means the imaginative numbers,” Teplotaxl explained hurriedly, and then he apologized to the eccentric gentlemen for having interrupted their labors.

And so it went. They looked in on Bonacci, but his room was teeming with rabbits; they passed rooms where Mayans and Arabs and Persians and Indians were working and talking and sleeping. The farther they went, the older the people seemed to be.

“The one over there, the one who looks like a maharajah,” said Teplotaxl, “he’s at least two thousand years old.”

The rooms grew in size and splendor until finally Teplotaxl and Robert stood before a kind of temple.

“We’re not allowed in here,” said Teplotaxl. “The man in white you’ll see is so important that a little devil like me can’t even say *boo* to him. He’s from Greece, and you wouldn’t believe the things he’s discovered! See those tiles on the floor? The

ones with the pentagons and stars? Well, one day he decided to cover the floor with them and when he couldn’t do it without leaving gaps between the tiles he came up with unreasonable numbers to explain why. Rutabagas, remember? The rutabaga of two, the rutabaga of five. And the blankety-blank numbers. You remember those, of course.”

“Oh, yes,” Robert assured him.

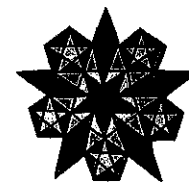
“Well, Pythagoras is the man’s name,” the number devil whispered. “You know what else he came up with? The word *mathematics*! Anyway, here we go.”

The hall they now entered was the largest Robert had ever seen. It was bigger than a gym, bigger than a cathedral, and much, much more beautiful. The walls were decorated with mosaics of the most varied patterns, and a majestic throne of gold stood at the first landing on a gigantic flight of steps that led so high there was no telling where it ended.

Robert couldn’t get over it. He had never dreamed the number devil lived in such luxury.

“Number Hell, my foot! This is Paradise!”

“Don’t be so sure. Oh, I can’t complain, but there are times late at night when I’m getting nowhere with my problem and I think I’ll go out of my mind! I’m only one step away from the



solution but a wall has grown up to keep me from it. That's hell!"

Robert tactfully held his tongue and looked around. Only then did he notice the endlessly long table in the middle of the hall and the waiters along the walls. Suddenly a beanpole of a man at the entrance swung a stick as far back as it would go and struck a gong, which resounded all through the palace.

"Follow me," said Teplotaxl. "Our places are at this end of the table."

Once they had taken their seats, they watched the more famous number devils file past. Robert recognized the Owl and Professor Horrors and then Bonacci (from the rabbit on his shoulder), but most of them he had never seen before. There were solemn Egyptians, there were Indians with pink dots on their foreheads, there were Arabs wearing burnouses and monks in habits, there were Africans and American Indians, Turks with curved swords, Americans in jeans. There were thousands of them.



Robert was amazed at how many number devils there were, but also at how few women he saw among them. He spotted no more than six or seven, and no one seemed to be taking them seriously.

"Why aren't there more women?" he asked. "Is there any rule against them?"

"They used to have a hard time of it. Palace policy was clear: Mathematics is man's work. But things seem to be changing of late."

When the guests had finally taken their seats and mumbled their greetings, the beanpole struck his gong again and the hall fell silent. A Chinese gentleman in fine silk raiment ascended first the stairs and then the throne.

"Who is that?" Robert asked.

"Could be the man who invented zero," Teplotaxl whispered.

"Is he the greatest of them all?"

"The second-greatest," said Teplotaxl. "The greatest lives up where the stairs lead, in the clouds."

"Is he Chinese too?"

"Nobody knows. None of us has ever seen him face-to-face. But we revere him greatly. He is commander-in-chief of all number devils, the man who discovered one. Though for all we know he may not be a man at all. He may be a woman!"



Robert was so impressed that he could say nothing.

Meanwhile the waiters had begun to serve the meal.

"Hey, they're starting with dessert!" Robert said, when a waiter placed a slice of pie on his plate.

"Sh! Not so loud, my boy. We eat nothing but pies, because pies are round and the circle is the most perfect of all figures. Here, try one."

Robert had never tasted anything so delicious in all his life.

"Supposing you want to find out how big the pie is," the number devil said. "How would you go about it?"

"Don't know. You've never shown me how and in school we're still on pretzels."

"What you need is an unreasonable number, the most important of them all. A man all the way up at the head of the table discovered it more than two thousand years ago. Another Greek. Without it we wouldn't know, to this day, how big a pie like this is—or how big our wheels or rings or oil tanks are. In other words, anything that's round. Even the moon and our very own earth. Without the number *pi* we'd be lost."

By this time the room was abuzz with number

devils having a good time, though here and there Robert saw one staring into space and another one making balls out of pie dough. Otherwise they all ate heartily and drank heartily (from pentagonal crystal glasses, fortunately, and not Dr. Little's weird bottles).

When the repast was over, the beanpole sounded the gong again and the man who may have discovered zero rose from his throne and disappeared up the stairs. Gradually the other number devils stood as well—the most eminent first, of course—and started back to their rooms. Soon only Robert and his protector were left.

Just as they were about to go, a gentleman in a magnificent uniform came up to them.

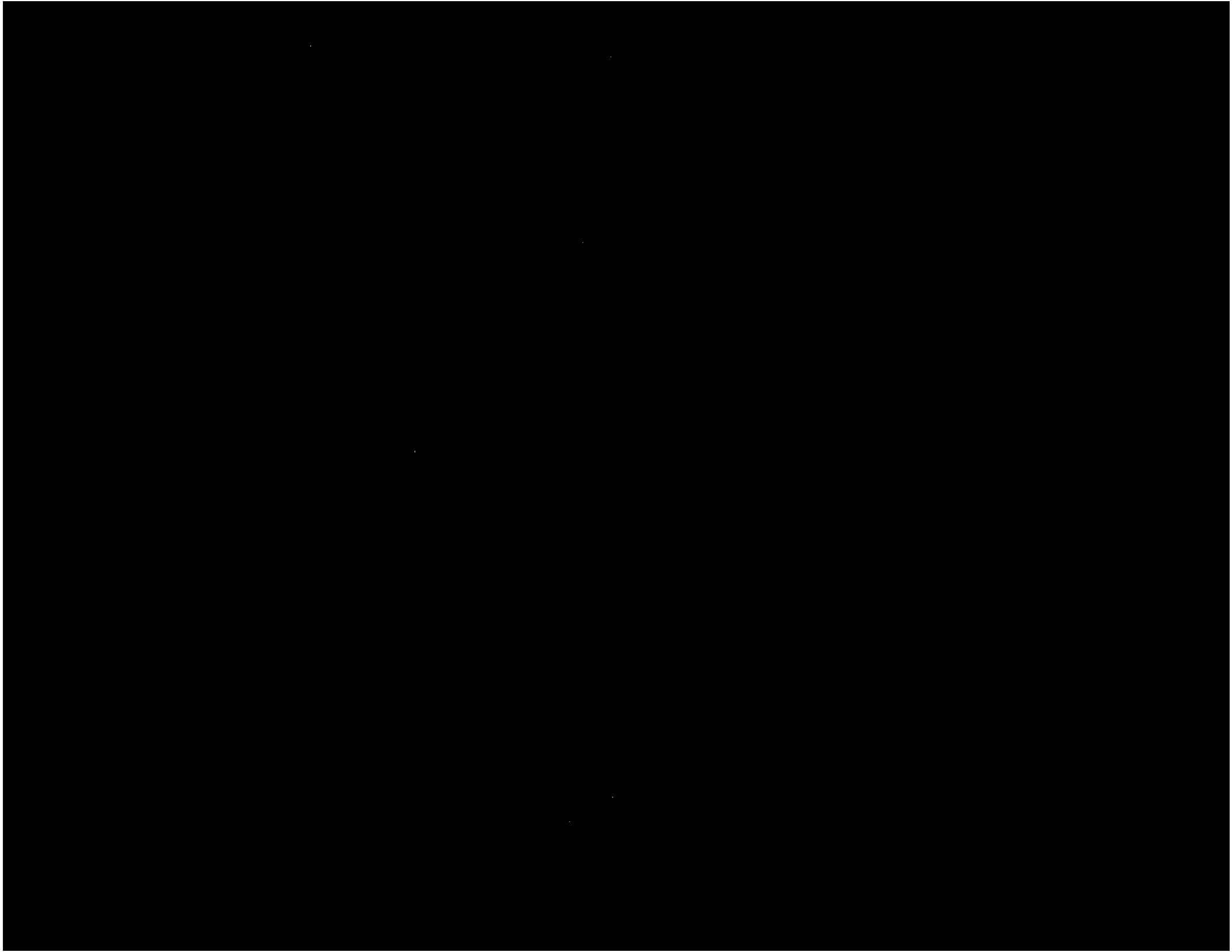
He must be the General Secretary, Robert thought, the man who signed his invitation.

"So this is your apprentice," said the dignitary in a sober voice. "Rather young, don't you think? He hasn't done any conjuring on his own yet, has he?"

"Not yet," Robert's friend replied. "But it won't be long now. Not at the rate he's going."

"How is he doing with prima-donna numbers? Does he know how many there are?"

"Precisely as many as there are ordinary and odd and hopping numbers," said Robert quickly.



“Very good. He can skip the examination. What is his name?”

“Robert.”

“Stand, Robert. By the power vested in me as General Secretary, I hereby accept you into the lowest rank of number apprentices and bestow upon you in recognition thereof the Order of Pythagoras, Fifth Class.”

With these words he ceremoniously hung a heavy golden chain with a five-pointed star around Robert’s neck.

“Thank you,” said Robert.

“It goes without saying that this distinction shall remain a secret,” the General Secretary added and, without so much as a nod, turned on his heel and disappeared.

“Well, that’s that,” said Robert’s friend and master. “I must be going. You’re on your own now.”

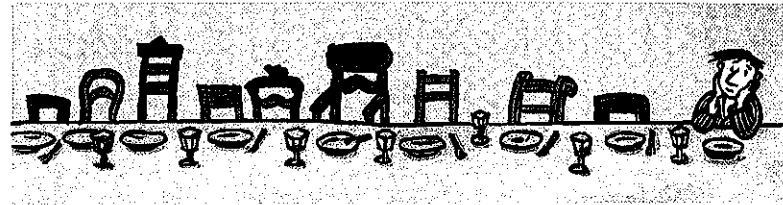
“What?” cried Robert. “You can’t leave me like that!”

“Sorry,” said Teplotaxl, “but I have my own work to attend to.”

Robert saw that Teplotaxl was moved. Robert was too. On the brink of tears, in fact: he hadn’t realized how much a part of his life the number devil had become. But neither the one nor the

other felt it seemly to show his emotions, so all Teplotaxl said was “Good-bye, Robert,” and all Robert said was “Bye.”

In a twinkling his friend was gone.



Robert was now all alone at the gigantic table. How in heaven’s name was he going to get home? he wondered. He felt the chain weighing heavier around his neck and the delicious pie growing heavier in his stomach, and before long he had nodded off and was soon so fast asleep that for all he knew he had never left his room on his master’s shoulders.

He awoke in his bed, of course, with his mother shaking him and saying, “Time to get up, Robert. If you don’t get up this very instant you’ll be late for school.”

The same as always, Robert said to himself. In your dreams you get delicious pies to eat, you may even get a chain with a star hung around your

neck, and the minute you open your eyes everything's back to normal.

But as he stood in front of the mirror in his pajamas, brushing his teeth, he felt something tickling his chest and looked down to see a tiny five-pointed star on a thin golden chain. He couldn't believe his eyes. This time his dream had come true!

After he'd dressed, he took the chain off and stuck it in his pocket: he didn't want his mother asking silly questions.

Where did that star come from? she'd want to know the minute she saw it. Boys don't wear jewelry!

How could he tell her it was the emblem of a secret order?

Things were normal in school except that Mr. Bockel looked more tired than usual and almost immediately took cover behind the newspaper to eat his pretzels in peace. But first he set forth a problem he was sure would take the whole hour to solve.

"How many of you are there in the class?" he asked, and the eager-beaver Doris shot up her hand and said, "Thirty-eight."

"Good, Doris. Now I want you to pay close attention. If the first boy down here in the front—

what's your name again? oh yes, Albert—well, if Albert gets one pretzel, and Betsy gets two pretzels, and Charlie three, Doris four, Enrique five, Felice six, and so on all the way up to number thirty-eight back there, how many pretzels do I need to buy to supply the entire class?"

Robert was incensed.

Mr. Bockel and his mind-bockeling problems! He thought, Now we'll have to work like the devil while he takes it easy.

Robert didn't let on how annoyed he was at the situation, but while all his classmates hunched over their notebooks he simply stared out into thin air.

"What's the matter, Robert?" Mr. Bockel asked. "Dreaming again?"

So he does keep one eye on the class, said Robert to himself.

"No, no," he said aloud. "I'm working on your problem." And he started writing:

$$1 + 2 + 3 + 4 + 5 + 6 \dots$$

God, how boring! By the time he got to eleven he was totally confused. He, the bearer of the

Order of Pythagoras (even if it was only fifth class)! But then he realized he wasn't wearing the star. He'd left it in his pocket.

Cautiously—he didn't want Mr. Bockel to see—he hung it around his neck under his shirt, and in a flash he had the most elegant solution to the problem. Triangle numbers. What else! It was as if he had learned them just for the occasion. How did they go now?

He wrote the following:

$$\begin{array}{r}
 1 \ 2 \ 3 \ 4 \ 5 \ 6 \\
 12 \ 11 \ 10 \ 9 \ 8 \ 7 \\
 \hline
 13 \ 13 \ 13 \ 13 \ 13 \ 13
 \end{array}$$

$$6 \times 13 = 78$$

If it worked for the numbers from one to twelve, it would work for one to thirty-eight!

$$\begin{array}{r}
 1 \ 2 \ 3 \ \dots \ 18 \ 19 \\
 38 \ 37 \ 36 \ \dots \ 21 \ 20 \\
 \hline
 39 \ 39 \ 39 \ \dots \ 39 \ 39
 \end{array}$$

$$19 \times 39 = ?$$

Cautiously again he pulled the calculator out of his briefcase and entered the following under the desk:

$$19 \times 39 = 741$$

“Got it!” he cried. “Nothing to it!”

“What?” said Mr. Bockel, letting his newspaper drop. “Well, what’s the answer?”

“741,” said Robert softly.

You could have heard a pin drop.

“How did you do it?” Mr. Bockel asked.

“Oh,” said Robert, clutching the star under his shirt, “it practically solved itself.” And he silently thanked the number devil for all he had done for him.





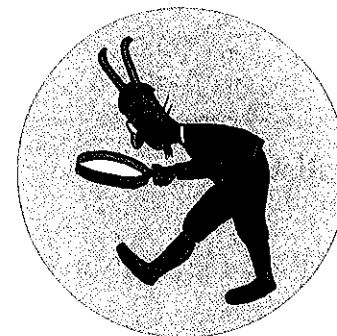
Warning!

Nothing in a dream is quite as it is in school or books. When Robert and the number devil talk about mathematics, they use some unusual expressions. No wonder. *The Number Devil* is anything but a usual story.

So don't think the dream words they use will be understood by everyone. Your mathematics teacher, for instance, or your parents. Mention "hopping" or "rutabagas" to them, and they won't know what you're talking about. Grownups have different words for such things: when they make a number hop twice, they square it or raise it to the power of two; they don't take the rutabaga, they take the square root. In mathematics classes "prima-donna numbers" are called prime numbers, "imaginative numbers" imaginary numbers, and you will never hear your teachers say "five vroom!" because the way they learned to say it from their teachers is five factorial.

Technical terms don't exist in dreams. Nobody dreams in big words. So when the number devil talks in images and sets numbers hopping instead of raising them to power of x or y , it's not just kid stuff. In dreams we do as we please.

In class, however, we never sleep and seldom dream. Your teacher is right to use the expressions used by mathematicians the world over. Please do the same (or you may get into trouble).



Seek-and-Ye-Shall-Find List

Once you've finished *The Number Devil*, you may not remember whether something you need is in it. The following list will help you to locate it if it is.

The list contains the dream words used by Robert and the number devil as well as the "correct," "official" terms, the ones mathematicians use. The latter appear in ordinary type, the dream words in *italics*.

The list also contains several expressions not used in the book at all. You don't need to worry about them. I have included them in case *The Number Devil* falls into the hands of mathematics teachers or other grownups (who deserve a chuckle of their own, after all).

Archimedes (287–212 B.C.)	246
Arithmetic progressions	100–101, 251–53
Assumptions	216
Axioms	219–22

Bertrand's Postulate	62
<i>Bonacci</i>	108, 242, 244
(Leonardo da Pisa, a.k.a. Fibonacci)	
<i>Bonacci numbers</i>	108–21, 149–40, 194
(Fibonacci numbers, Fibonacci sequence)	
<i>Broom brigade</i> (Combination to size n)	161–67
Cantor dust	240
Cantor, Georg (1845–1918)	240–41
(Professor Singer)	
<i>Changing places</i> (Permutation)	149–57
Circle measurement (<i>Pies</i>)	246
<i>Coconuts</i> (Triangular numbers)	93–101
Combination to size n	161–67
(<i>Broom brigade</i>)	
Combination without repetition	158–61
(<i>Handshakes</i>)	
Combinatorics	149–67
Continued fractions	198
Countable infinite sets	16–17, 173–79
Cube (Hexahedron)	205, 207
Decimal fractions	68–75
Decimal fractions, nonperiodic	68–75, 195
Decimal fractions, periodic	75, 195

Decimal system	32–44
Diagonals	80–81
Dividing	50–51
Dividing by zero	54–56
Dodecahedron (<i>Pentagon ball</i>)	206
<i>Dots</i> (Vertices)	202–4, 207–10
<i>Double pyramid</i> (Octahedron)	205
Edges (<i>Lines</i>)	202–4, 207–10
Eratosthenes (c. 280–200 B.C.)	57–60
Euler, Leonhard (1707–1783) (<i>Owl</i>)	241–42, 244
Euler's formulas	202–4, 210
Factor analysis	61
Factorial (<i>Vroom!</i>)	156–57, 179, 228
Fibonacci numbers,	108–21, 139–40
Fibonacci sequence (<i>Bonacci numbers</i>)	
Fractals	191–93
Fractions	18–20, 181–87
<i>Garden-variety numbers</i>	55, 173–75
(Natural numbers)	
Gauss, Carl Friedrich (1777–1855)	241–42, 244
(<i>Professor Horrors</i>)	
Geometric series	181–83

Goldbach's conjecture	62-63
Golden mean	196-200
<i>Gum division</i>	17-20
(Infinitely small numbers)	
<i>Gum multiplication</i>	16
(Infinitely large numbers)	
<i>Handshakes</i> (Combination without repetition)	158-61
Harmonic series	184-87
Hexahedron (Cube)	205, 207
<i>Hopping</i> (Raising to a higher power)	38-41
<i>Hopping backward</i> (Taking the root)	76
<i>Hopping twice</i> (Squaring)	79-80, 136, 179
<i>Horrors, Professor</i>	241-42, 244
(Carl Friedrich Gauss)	
$i(\sqrt{-1})$	242
Icosahedron	205-6
Imaginary numbers (<i>Imaginative numbers</i>)	242
<i>Imaginative numbers</i> (Imaginary numbers)	242
Infinitely large numbers	16-17
Infinitely small numbers	17-20
Irrational numbers	75, 78, 195
(<i>Unreasonable numbers</i>)	

Klein, Felix (1849-1925)	239-40
(<i>Dr. Happy Little</i>)	
Klein bottle (<i>Little bottle</i>)	239-40, 247
Koch curve	192
Leonardo da Pisa, a.k.a. Fibonacci	108, 242, 244
(c. 1170-1240) (<i>Bonacci</i>)	
Limit	71, 183, 196
<i>Lines</i> (Edges)	202-4, 207-10
<i>Little, Dr. Happy</i> (Felix Klein)	239-40, 247
<i>Little bottle</i> (Klein bottle)	239-40, 247
Lune, Johan van de (<i>Man in the Moon</i>)	222-23
<i>Man in the Moon</i> (Johan van de Lune)	222-23
<i>Minus numbers</i> (Negative numbers)	34-35
Natural numbers	55, 173-75
(<i>Ordinary, garden-variety numbers</i>)	
Negative numbers (<i>Minus numbers</i>)	34-35
Networks	202-3
<i>Number triangle</i> (Pascal's triangle)	128-45
Octahedron (<i>Double pyramid</i>)	205
Odd numbers	174-75
One	15, 245

Optimization problem	229
<i>Ordinary numbers</i> (Natural numbers)	55, 173–75
<i>Owl</i> (Leonhard Euler)	241–42, 244
Pascal, Blaise (1623–1662)	128–45
Pascal's triangle (<i>Number triangle</i>)	128–45
Pentagon	199–202
<i>Pentagon ball</i> (Dodecahedron)	206
Permutation (<i>Changing places</i>)	149–57
Pi (π)	246
<i>Pies</i> (Circle measurement)	246
Polygons	203–10
Polyhedron	204
Polytopic surfaces	230
<i>Pretzels</i> (Topological objects)	239
<i>Prima-donna numbers</i> (Prime numbers)	55–63, 176
Prime-number testing	57–61
<i>Principia Mathematica</i> (Bertrand Russell and A. N. Whitehead)	224–25
Proofs	217–26, 229
<i>Pyramid ring</i>	209–10
Pyramids (Tetrahedron)	204
Pythagoras (sixth century B.C.)	242–43
Pythagorean theorem	81–83

<i>Quang</i>	81–82, 199
<i>Rabbit clock</i>	113–20
<i>Rabbits</i>	111–21
Raising to a higher power (<i>Hopping</i>)	38–41
Recursion	108, 195, 197
Roman numerals	33
Russell, Bertrand (1872–1970) (<i>Lord Rustle</i>)	224–26, 238–39
<i>Rustle, Lord</i> (Bertrand Russell)	224–26, 238–39
<i>Rutabagas</i> (Square roots)	76–80
Self-similarity	200
Sequences	174–76
Series	180–87
Sierpiński triangle	140–45
Sieve of Eratosthenes (Test for prime numbers)	57–60
Simple fractions	18–20, 182–83
Simplex algorithm	230
<i>Singer, Professor</i> (Georg Cantor)	240–41
<i>Snowflakes</i>	191–92
Square numbers	102–3
Square roots (<i>Rutabagas</i>)	76–80
Squaring (<i>Hopping twice</i>)	79–80, 136, 179

Taking the root (<i>Hopping backward</i>)	76
Taking the square root (<i>Taking the rutabaga</i>)	76–80
Tetrahedron (Pyramids)	204
Tetrahedron ring	209–10
Topological objects	239
<i>Tour problem</i> (Traveling salesman problem)	226–29
Traveling salesman problem (<i>Tour problem</i>)	226–29
<i>Tree</i>	121–22
<i>Triangle numbers</i> 93, 101, 133–35, 159–61, 252–53 (Triangular numbers)	
Uncountable sets	83–84, 241
<i>Unreasonable numbers</i> (Irrational numbers)	75, 78, 195
Vertices (<i>Dots</i>)	202–4, 207–10
<i>Vroom!</i> (Factorial)	156–57, 179, 228
Zero	32–36, 41, 245
Zero power	218

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