What is the relationship between the maximum thickness of an airfoil and the lift force generated by it?

I. Introduction

This investigation aims to find the relationship between the maximum thickness of an aerofoil and the lift force generated. The topic is of significance because there have been large amounts of literature on the angle of attack and the lift force generated but not necessarily on the thickness of an aerofoil. To me it is important because I am very interested in aeronautics; I aim to fly ULMs first and then planes perhaps. I have seen vast amounts of literature on the lift force and the angle of attack, but I was interested by the effects of increasing the thickness of an aerofoil on the lift force. There has not been so much research done exactly on the relationship between thickness and lift force, at least from the resources I have consulted, so it was even more exciting for me to carry out this project.

A *The introduction includes a clear and explicit description of the aim, as well as a rationale. The plan of action is not explicitly explained.*

B *Constraints relative to this specific study are appropriately defined.*

C *The Mathematical exploration relates to personal interest.*

C *A specific model is chosen.*

The aerofoil of a standard ULM will be used to make the investigation more concrete. The wing is 0.8m wide, 8m long and with a variable height (thickness). A standard ULM has a one meter width, but this ULM being small; it will only be 0.8m wide. The height will range from 5cm to 25cm, above which the thickness is bigger than 30% of the chord line. It will be of the same shape, with a camber towards the tip, a smoother tail, and a straight lower side of the wing[1].

Low-speed ULM (1 m)

II. Theory

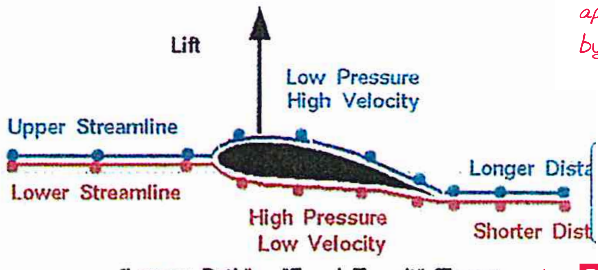
When the wind blows against a wing, which is essentially a solid obstructing the wind's path, an area of lower pressure is created on the upper side, while the lower side experiences a high pressure. This pressure difference triggers a force perpendicular to the fluid, from an area of high pressure to an area of low pressure, as displayed by the Figure 1. Moreover, the pressure at the left tip of the wing on the upper side is higher than the pressure more to the right. This in turn creates an imbalanced force from left to right, accelerating the speed of the wind above the wing, not below.

A/B *Useful explanation, appropriately supported by the diagram.*

Figure 1 – Diagram illustrating the theory[2]

The second important notion is Bernoulli's principle, which shows the inverse relationship between pressure P and velocity[3] v.

$$\frac{v^2}{2} + \frac{P}{\rho} = C$$



"Longer Path" or "Equal Transit" Theory

B *The variable P used in the formula is explained at this point. The definition (fluid density) is made at the next paragraph.*

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[1] URL of picture: <http://upload.wikimedia.org/wikipedia/commons/7/75/Examples_of_Airfoils.svg>

[2] Benson, Tom. *Equal Transit Theory* "National Aeronautics and Space Administration" NASA. July 2008. Web. 20 Dec. 2014. http://www.grc.nasa.gov/WWW/k-12/airplane/wrong1.html

[2]Alward, Joseph. "Aerodynamic Lift. Coanda Effect. Bernoulli's Equation. Angle of Attack." Aerodynamic Lift. Coanda Effect. Bernoulli's Equation. Angle of Attack. N.p., n.d. Web. 14 Dec. 2014. <http://www.aerodynamiclift.com/>.

Moreover[4]:

$$\Delta P = \frac{1}{2} \times \rho \times \Delta(v^2) \quad or \quad \Delta P = \frac{1}{2} \times \rho \times ({v_1}^2 - {v_2}^2)$$

E Research has been conducted on the literature of the domain of the study, in order to provide the equations relevant to the problem.

Where P is the pressure, ρ is the fluid density, which we assume will remain constant, v_1 is the wind speed under the wing and v_2 the wind speed above. Turning this pressure differential into force gives[5]

$$Pressure = \frac{Force}{Area} \Rightarrow Lift\ Force\ (F) = Pressure\ differential(\Delta P)\ \times Planform\ area(A)$$

B The (differential) Δ operator is not defined in the equations provided. It is implicitly explained at the next line.

If we combine the two equations, we have[6]:

$$F = \frac{1}{2} \times \rho \times ({v_1}^2 - {v_2}^2) \times A$$

To calculate the lift force, I have to find the velocity above and under the wing, and the planform area of the wing which is subject to the lift. The density of the fluid will be kept constant at a standard value of 1.22 $kg.m^{-3}$ and the velocity of the wind under the wing will be 8m.s^{-1}.
The distance travelled by the fluid above the wing is longer, but it does it in the same time, which means that it has a higher velocity than the fluid under the wing. The distance travelled will be x times larger with different heights, hence the velocity above the wing will also be x times larger than the constant velocity under the wing. This is how I will be able to calculate the velocity of the fluid above the wing at different points on the wing.

A Any particular reason for choosing these values?

A, B Interesting explanations on the method and process that will be used.

III. Calculation of the planform area

The planform area is the area subject to the lift force from an area of high pressure to an area of low pressure. On most aerofoils, the lower side would be curved as well, but on this one, the lower side is flat, so the planform area calculated here is relatively accurate.

I looked at shapes of wings on planes, and I saw that on ULMs, the wing is often curved at the end, surely for aerodynamic purposes. This is why it is moderately curved towards the end. As a result, I will have to use integration to calculate the area subject to the lift force. I am going to calculate the area in the intervals $x \in [0, 0.1]$, $x \in [0.1, 0.2]$, $x \in [0.2, 0.3]$, $x \in [0.3, 0.4]$, $x \in [0.4, 0.5]$, $x \in [0.5, 0.6]$, $x \in [0.6, 0.7]$, $x \in [0.7, 0.8]$. In that way I will be able to work out the lift force in the relevant areas, because each of those intervals will have a different distance between two points on the curve, hence a different velocity and a different pressure differential.

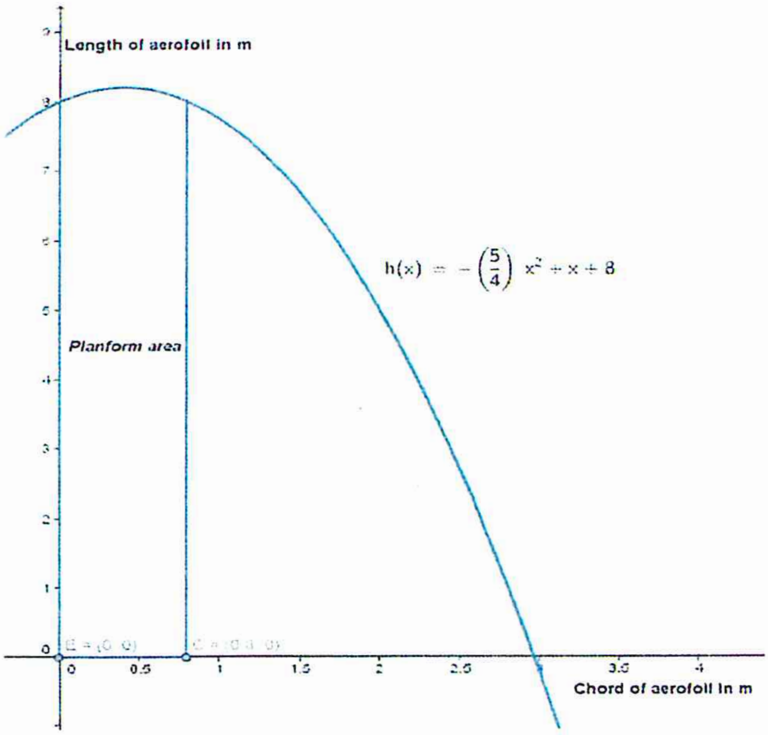
C Research has been conducted on different models and a choice is made.

E Very good understanding of the complexity of the modelling.

This picture shows what the planform area corresponds to, and the rounded edge[7]:

[4] Ibid.
[5] Ibid.
[6] URL of the picture:
http://www.evektoraircraft.com/modfiles/animation/images/big/OWYyYzEyYzAyNjAyZjEyOGM3ZDU2ZTU1OGJmZjk4YmM.jpg

Figure 2 – Graph illustrating the planform area.



B *How was this equation determined any particular reason?*

E *Appropriate use of mathematics to calculate the area under the curve.*

The equation of the curve is:

$$h(x) = -\frac{5}{4}x^2 + x + 8$$

Consider S the indefinite integral of the planform area:

$$S = \int \left(-\frac{5}{4}x^2 + x + 8\right) dx$$

$$= -\frac{5}{12}x^3 + \frac{x^2}{2} + 8x + K$$

A *The task would benefit from relevant explanations, in order to explain the process to readers who are not familiar with the problem or mathematics involved.*

$$For\ x \in [0,0.1]\ \ I_1 = \left[-\frac{5}{12}x^3 + \frac{x^2}{2} + 8x + K\right]_0^{0.1} = 0.805\ m^2$$

B *The results provided are correct to three significant figures.*

$$For\ x \in [0.1,0.2]\ \ I_2 = \left[-\frac{5}{12}x^3 + \frac{x^2}{2} + 8x + K\right]_{0.1}^{0.2} = 0.812\ m^2$$

$$For\ x \in [0.2,0.3]\ \ I_3 = \left[-\frac{5}{12}x^3 + \frac{x^2}{2} + 8x + K\right]_{0.2}^{0.3} = 0.817\ m^2$$

$$For\ x \in [0.3,0.4] I_4 = \left[-\frac{5}{12}x^3 + \frac{x^2}{2} + 8x + K\right]_{0.3}^{0.4} = 0.820\ m^2$$

The function h(x) is quadratic, so with one turning point, which means that it is also the axis of symmetry of the function.

E *The vertex lines on the axis of symmetry, a point cannot be an axis of symmetry (improper terminology).*

$$h(x) = -\frac{5}{4}x^2 + x + 8\ \ so\ \ h'(x) = -\frac{5}{2}x + 1$$

$h'(0.4) = 0$ and $h(0.4) = 8.2$ so the maximum point P of $h(x)$ has coordinates $(0.4, 8.2)$.

Moreover, the function $h'(x)$ is a linear equation, so there is only one solution where the gradient of $h(x) = 0$, hence only one turning point. Therefore P is the vertex of $h(x)$ and the areas in the next intervals will be equal to the areas in the intervals preceding P.

A, B, C *Although the explanations are not correctly expressed, the symmetry of the quadratic function is used in order to avoid repetition of calculations. This helps making the work concise.*

$Area\ for\ x \in [0.4,0.5]\ \leftrightarrow\ I_4 = 0.820\ m^2$ $\quad$ $Area\ for\ x \in [0.5,0.6]\ \leftrightarrow\ I_3 = 0.817\ m^2$

$Area\ for\ x \in [0.6,0.7]\ \leftrightarrow\ I_2 = 0.812\ m^2$ $\quad$ $Area\ for\ x \in [0.7,0.8]\ \leftrightarrow\ I_1 = 0.805 m^2$

IV. Calculation of the velocity above the wing for different values of maximum height *h*

1. *Method*

As stated in Part II, I will compare the distance travelled by the wind on the upper side and the lower side in the same amount of time, which will give me a value for the velocity at different parts of the wing. I plotted 4 points on an aerofoil to find a function for the upper side of the aerofoil, which I will then use to find the distance between relevant points. The shape I have modelled is similar to a ULM aerofoil with a flat lower side and a cambered upper side (see Figure 1). I plotted four points as displayed on the graph (A, B, C and D) on the image of a low speed ULM aerofoil and from there I could derive a system of simultaneous equations to yield a function.

C *Personal example.*

A *Appropriate explanations.*

The function $f(x)$goes through four points: $A\ (0,0), B\ (0.83, 0.66), C\ (6.41, 0.51)$ and $D\ (8,0)$.

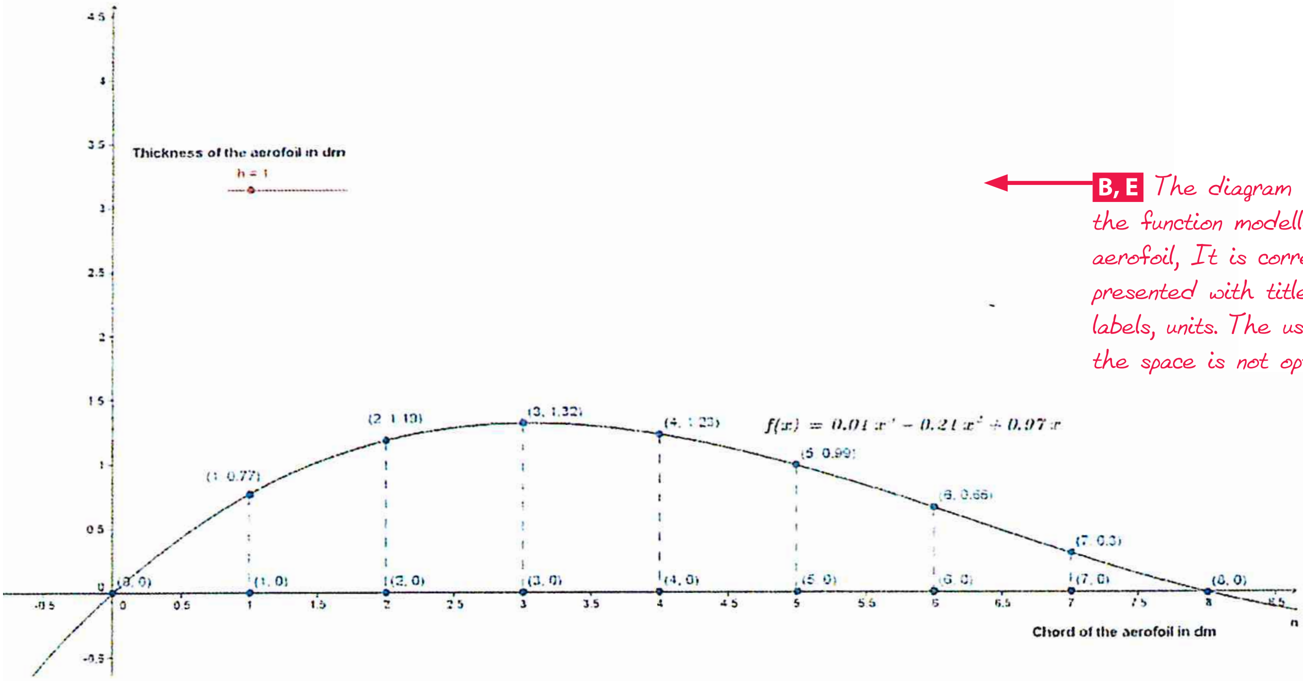
Consider this system of four simultaneous equations:

$$\begin{array}{l} L_1 \\ L_2 \\ L_3 \\ L_4 \end{array} \begin{cases} f(0) = 0 \\ f(0.83) = 0.66 \\ f(6.41) = 0.51 \\ f(8) = 0 \end{cases} \Leftrightarrow \begin{cases} a \times 0 + b \times 0 + c \times 0 + d = 0 \Rightarrow d = 0 \\ a \times 0.83^3 + b \times 0.83^2 + c \times 0.83 + 0 = 0.66 \\ a \times 6.41^3 + b \times 6.41^2 + c \times 6.41 + 0 = 0.51 \\ a \times 8^3 + b \times 8^2 + c \times 8 + 0 = 0 \end{cases}$$

This system yields $\begin{cases} a = 0.01 \\ b = -0.21 \\ c = 0.97 \\ d = 0 \end{cases}$ to 2 decimal places. Hence $f(x) = 0.01x^3 - 0.21x^2 + 0.97x$

B, E *Technology is used in order to determine this function.*

Figure 5 – Graph showing F(x) when h=1 and the intervals for the calculation of the velocity.



B, E *The diagram displays the function modelling the aerofoil, It is correctly presented with title, labels, units. The use of the space is not optimal.*

On the graph, f(x) is the upper side. The velocity on the upper side will be calculated in 8 parts:
$x \in [0,1],\ x \in [1,2],\ x \in [2,3], x \in [3,4], x \in [4,5], x \in [5,6], x \in [6,7], x \in [7,8]$.

The function f(x) will be vertically stretched by a factor h (the slider h on the graph) to obtain various values of the maximum thickness. Moreover, the formula I am using to calculate the distance between two points on a curve[8]:

E *Interesting idea.*

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Where y is the function $F(x)$ or $h \times f(x) \leftrightarrow F(x) = h(0.01x^3 - 0.21x^2 + 0.97x)$

E *Appropriate use of mathematics, beyond the scope of the Math HL syllabus.*

2. *Finding the right values of h to use on the curve.*

First I am going to find the turning points' x-coordinates as h changes:

$$\frac{dF(x)}{dx} = h(0.03x^2 - 0.42x + 0.97) = 0$$

$$x = \frac{0.42 \pm \sqrt{0.06}}{0.06} = 7 \pm \frac{\sqrt{0.06}}{0.06}$$

$$x_1 = 11.08 \quad x_2 = 2.92$$

Finding the second derivative of F(x):

$$\frac{dF(x)^2}{d^2x} = h \times (0.06x - 0.42)$$

For x_1 , $F''(11.08) = 0.245h$ but h > 0 so it will always be positive and x_1 is hence the minimum point. For x_2 , $F''(2.92) = -0.275h$ but h > 0 so it will always be negative and x_2 is therefore the maximum point, which we are interested it in. The y-coordinate of the maximum point, in other words, the thickness of the aerofoil which we are investigating as h changes:

E *Correct use of calculus.*

$$Thickness = h \times (1.29) = 1.29 \times h\ dm\ or\ 12.9 \times h\ cm.$$

Maximum thickness in dm	Percentage of chord in %	Value of h to 2 d.p.
0.5	6	0.39
0.7	9	0.54
0.9	11	0.70
1.1	14	0.85
1.3	16	1.00
1.5	19	1.16
1.7	21	1.32
1.9	24	1.47
2.1	26	1.63
2.3	29	1.78
2.5	31	1.93

B *The data presented in table are expressed in decimetre. This value (5cm) corresponds to the first row.*

The first maximum thickness is 5cm because below this, no notable change would be made in the velocity of the wind on the upper side. Most aerofoils made for planes have a maximum thickness that is about 20% of the chord, for example, the airfoil NACA 16-123 has maximum thickness 23% of chord. Some aerofoils are 30% thick and over, but there are mostly made for wind turbines not planes, such as Althaus AH 93-W-300[9]. I could explain this more thoroughly using the drag force. The main point is that at this kind of thickness, such a high drag force would be generated that the lift force would be insignificant in comparison. However, the drag force is not part of this investigation.

A *Extended research has been conducted.*

D *Critical reflection on the research made and its applications to the current problem.*

E *Good understanding of the subject is demonstrated.*

[8] Miguel Lerma. *Arc Length, Parametric Curves* "Math 214-2 Integral Calculus". Miguel Lerma Fall2004. <http://www.math.northwestern.edu/~mlerma/notes/c2-arclength.pdf>

[9] "Airfoil Tools." Airfoil Tools. N.p., 2014. Web. 11 Jan. 2014. <http://airfoiltools.com/>

3. *Finding the distances on the curve F(x)*

a. $\frac{dF(x)}{dx} = h(0.03x^2 - 0.42x + 0.97)$

b. Finding the square of $\frac{dF(x)}{dx}$

$$\left(\frac{dF(x)}{dx}\right)^2 = h^2 \times \left(\frac{df(x)}{dx}\right)^2$$

$$\left(\frac{df(x)}{dx}\right)^2 = h^2 \times (0.0009x^4 - 0.0126x^3 + 0.0291x^2 - 0.0126x^3 + 0.1764x^2 - 0.4074x + 0.0291x^2 - 0.4074x + 0.9409)$$

$$\left(\frac{dF(x)}{dx}\right)^2 = h^2(0.0009x^4 - 0.0252x^3 + 0.2346x^2 - 0.8148x + 0.9409)$$

c. Integrating for a and b in terms of h

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$L = \int_a^b \sqrt{1 + h^2(0.0009x^4 - 0.0252x^3 + 0.2346x^2 - 0.8148x + 0.9409)}$$

Here I used GDC to calculate the area under the curve, for each value of h and each interval. (Elliptic integrals are not at all in the scope of our Math HL syllabus).

B *Good use of technology, in order to generate (correct) data.*

Value of h	Distance between two points on the curve $F(x)$ in dm (to 3d.p.) in the interval...							
	$x \in [0,1]$	$x \in [1,2]$	$x \in [2,3]$	$x \in [3,4]$	$x \in [4,5]$	$x \in [5,6]$	$x \in [6,7]$	$x \in [7,8]$
0.39	1.045	1.013	1.001	1.002	1.007	1.014	1.018	1.018
0.54	1.085	1.025	1.003	1.003	1.014	1.027	1.034	1.034
0.70	1.138	1.042	1.004	1.005	1.024	1.045	1.057	1.057
0.85	1.198	1.062	1.007	1.007	1.035	1.065	1.083	1.083
1.00	1.265	1.084	1.009	1.010	1.048	1.089	1.114	1.114
1.16	1.344	1.112	1.012	1.014	1.064	1.118	1.150	1.150
1.32	1.430	1.142	1.016	1.018	1.082	1.150	1.191	1.191
1.47	1.514	1.174	1.019	1.022	1.100	1.184	1.233	1.233
1.63	1.609	1.210	1.024	1.027	1.122	1.222	1.280	1.280
1.78	1.700	1.246	1.028	1.032	1.144	1.260	1.327	1.327
1.93	1.795	1.283	1.033	1.037	1.168	1.300	1.376	1.376

4. *Finding the velocity on the upper side*

While the wind on the upper side travelled the distances above, the wind on the lower side travelled 0.1m at 8m/s. The fact that $v = \frac{ds}{dt}$ means the ratio between the distance above the wing and under the wing will be equal to the ratio of the velocities above and under.

Therefore, to find the velocity in the relevant intervals at different values of h, I can use:

E Good reasoning, supported by appropriate explanations

$$Velocity\ above\ the\ wing\ (v_2) = Velocity\ under\ the\ wing\ (8ms^{-1}) \times \frac{Distance\ above\ in\ dm}{Distance\ under\ in\ dm}$$

$$v_2 = 8 \times \frac{Distance\ on\ the\ curve\ h(x)}{1}$$

Value of h	Velocity above the wing in ms⁻¹ to 3 d.p							
	$x \in [0,1]$	$x \in [1,2]$	$x \in [2,3]$	$x \in [3,4]$	$x \in [4,5]$	$x \in [5,6]$	$x \in [6,7]$	$x \in [7,8]$
0.39	8.360	8.104	8.008	8.016	8.056	8.112	8.144	8.144
0.54	8.680	8.200	8.024	8.024	8.112	8.216	8.272	8.272
0.7	9.104	8.336	8.032	8.040	8.192	8.360	8.456	8.456
0.85	9.584	8.496	8.056	8.056	8.280	8.520	8.664	8.664
1	10.120	8.672	8.072	8.080	8.384	8.712	8.912	8.912
1.16	10.752	8.896	8.096	8.112	8.512	8.944	9.200	9.200
1.32	11.440	9.136	8.128	8.144	8.656	9.200	9.528	9.528
1.47	12.112	9.392	8.152	8.176	8.800	9.472	9.864	9.864
1.63	12.872	9.680	8.192	8.216	8.976	9.776	10.240	10.240
1.78	13.600	9.968	8.224	8.256	9.152	10.080	10.616	10.616
1.93	14.360	10.264	8.264	8.296	9.344	10.400	11.008	11.008

B, E Correct and appropriate data are generated.

V. Calculation of the Lift Force in intervals at different thicknesses

Recalling from Section I, the lift force can be calculated thus:

$$F = \frac{1}{2} \times \rho \times (v_1{}^2 - v_2{}^2) \times A$$

- The planform area was determined already:

Interval of x in dm	Area in m² to 3 d.p.
$x \in [0,1]$	0.805
$x \in [1,2]$	0.812
$x \in [2,3]$	0.817
$x \in [3,4]$	0.820
$x \in [4,5]$	0.820
$x \in [5,6]$	0.817
$x \in [6,7]$	0.812
$x \in [7,8]$	0.805

- v_1 is 8m.s⁻¹
- v_2 is in the table above – the velocity above the wing.
- I will take the value of air density as 1.22 $kg.m^{-3}$

With all the components determined, I can determine the lift force for each thickness and in every interval:

A, B, E *Final results for the drag force are produced. The lines of reasoning are consistent and easy to follow through.*

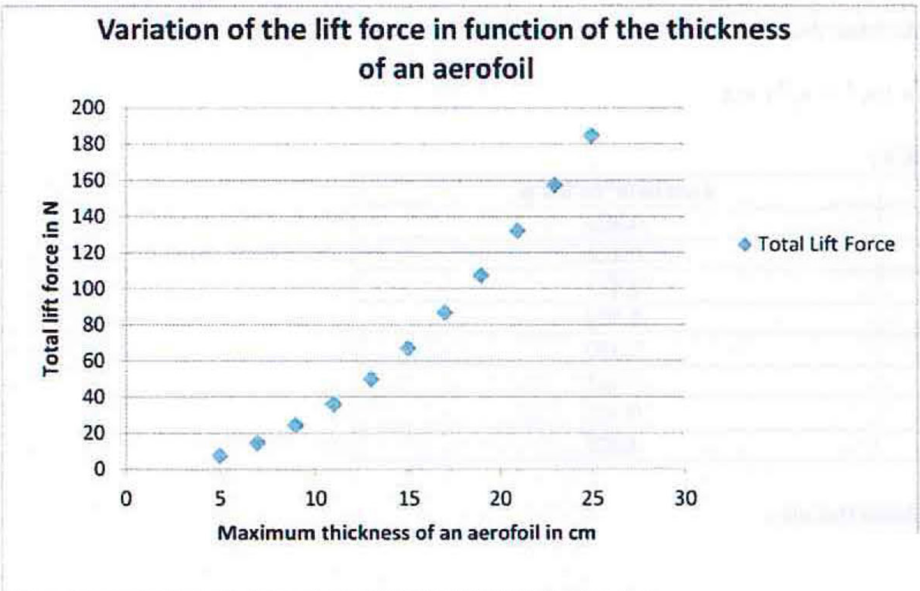
Value of h	Lift force in N to 2 d.p.							
	$x \in [0,1]$	$x \in [1,2]$	$x \in [2,3]$	$x \in [3,4]$	$x \in [4,5]$	$x \in [5,6]$	$x \in [6,7]$	$x \in [7,8]$
0.39	2.89	0.83	0.06	0.13	0.45	0.90	1.15	1.14
0.54	5.57	1.60	0.19	0.19	0.90	1.75	2.19	2.17
0.70	9.27	2.72	0.25	0.32	1.56	2.94	3.72	3.68
0.85	13.68	4.05	0.45	0.45	2.28	4.28	5.48	5.43
1.00	18.86	5.55	0.57	0.64	3.15	5.93	7.64	7.57
1.16	25.34	7.50	0.77	0.90	4.23	7.97	10.22	10.14
1.32	32.84	9.64	1.02	1.16	5.47	10.29	13.27	13.15
1.47	40.61	12.00	1.22	1.42	6.72	12.82	16.49	16.35
1.63	49.93	14.71	1.54	1.75	8.29	15.73	20.24	20.06
1.78	59.4	17.51	1.80	2.08	9.88	18.74	24.12	23.91
1.93	69.83	20.48	2.13	2.41	11.66	22.01	28.32	28.08

Here I have added up the lift forces in each interval to know the overall lift force provided by the aerofoil at each thickness:

Maximum thickness in cm	Total lift force in N to 2 d.p.
5	7.56
7	14.57
9	24.46
11	36.10
13	49.92
15	67.07
17	86.84
19	107.63
21	132.26
23	157.46
25	184.92

Figure 6 – Graph showing the relationship between Lift Force and the thickness of the aerofoil.

B *Appropriate display of the results.*



The relationship is non-linear so must be manipulated in order to find a mathematical relationship.

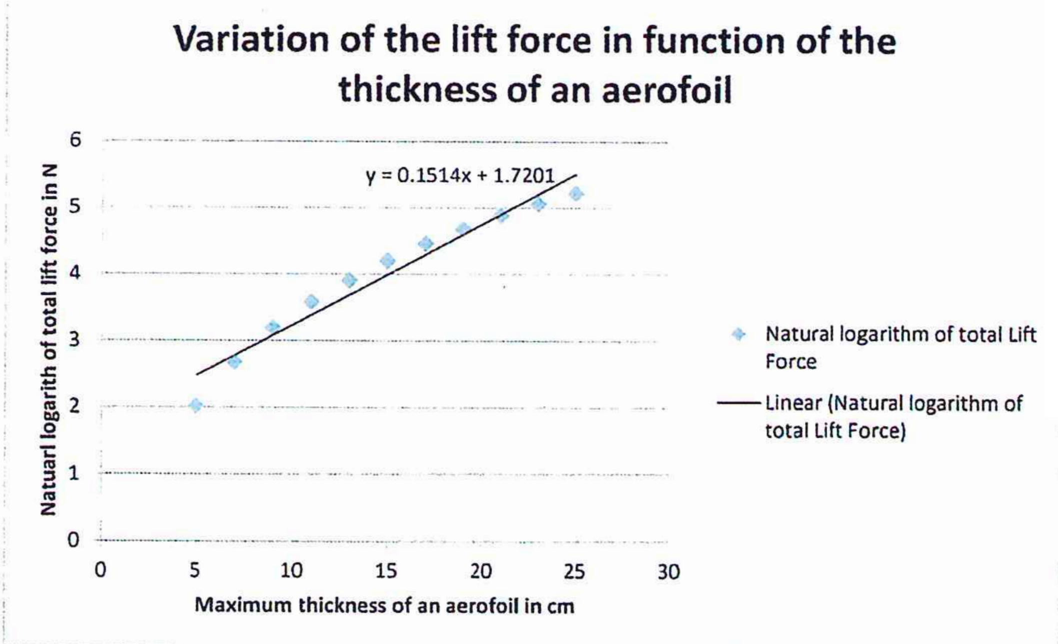
$y = a \times e^{b\times x}$ where a and b are to be determined, y is the lift force and x is the thickness

Then $\frac{y}{a} = e^{b\times x}$ *so* $\ln\left(\frac{y}{a}\right) = \ln\left(e^{b\times x}\right)$ and consequently $\ln(y) = b \times x + \ln(a)$

This is a linear equation of form $y = mx + c$ so plotting the graph of the natural logarithm of the lift force against the value for the maximum thickness yields a line of best fit whose equation can be used to find a and b.

E *The method used to find the model is similar to that used in Physics.*

Maximum thickness in cm	Total lift force in N to 2 d.p.	Natural logarithm of Total Lift Force in N to d.p.
5	7.56	2.02
7	14.57	2.68
9	24.46	3.20
11	36.10	3.59
13	49.92	3.91
15	67.07	4.21
17	86.84	4.46
19	107.63	4.68
21	132.26	4.89
23	157.46	5.06
25	184.92	5.22



B, E *The data manipulated are displayed and the linear relationship is visible. The unit of the y-axis is not properly expressed (confusing notation). The line of best fit is also displayed.*

Here, the equation of the line of best fit is: $y = 0.1514x + 1.7021$. Hence, the gradient of this line is equal to the value of b.

Also, the line crosses the y-axis at $y = 1.7021, x = 0$.

Therefore, $1.7021 = b \times 0 + \ln(a)$ and $a = e^{1.7021}$

$$\begin{cases} a = 5.59 \\ b = 0.15 \end{cases} \text{ to 2 decimal places}$$

$$\text{Total Lift Force in N: } L(x) = 5.59 \times e^{0.15x}$$

Where x is the maximum thickness of the aerofoil in cm.

B *...function...*

This equation could be useful in finding the optimum thickness of an aerofoil, while keeping the drag force lowest. Unfortunately, the drag force is not part of this investigation. I cannot be sure of the validity of the equation, it is most likely wrong; because of the small range of readings and the specificity of the aerofoil I have chosen. Yet, it gives an estimation of how the lift force varies as the aerofoil becomes thicker.

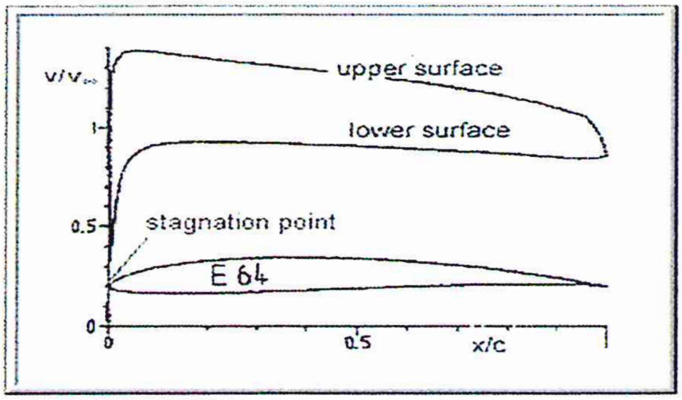
D *Critical analysis of the function obtained.*

VI. <u>Evaluation of the findings</u>

The lift force provided by the wings seems very small compared to the weight of the plane. First, this is only the force for one wing, but this would still not be enough. Most importantly, the Equal Transit theory underestimates the velocity on the upper side greatly though the key to the investigation is the change in velocity above the wing relative to the wind speed below. I have found a typical velocity distribution above the wing in relation to the wind speed.

D *Critical analysis of the results obtained for the lift force.*

Figure 7 – Standard velocity distribution[10].



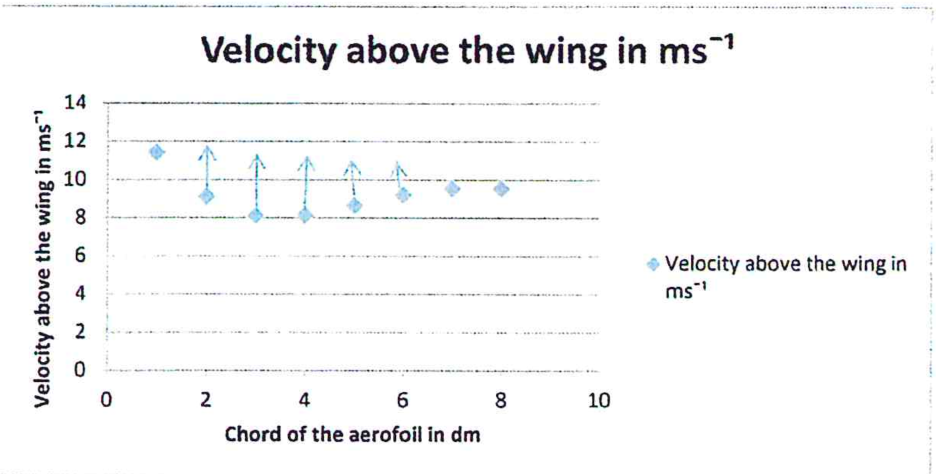
The y-axis show the ratio wind speed above: wind speed below, ranging from 1 to approximately 1.4. In my investigation, the findings range from 1 to 1.2 for a thickness of 17cm, up to 1 to 1.8 for a rather unrealistic thickness of 25cm. Roughly, the range of velocities I have found seems to fit.

D *Very good and interesting analysis of the results in the light of further research.*

[10] Hepperle, Martin. *Velocity and Pressure Distributions* "mh-aerotools" Martin Hepperle. May 2005. December 2013 <http://www.mh-aerotools.de/airfoils/velocitydistributions.htm>

However, the biggest difference lies in the shape of the velocity distribution across the chord of the aerofoil. The graph below is the velocity distribution of the typical values I have found. It represents the velocity distribution when the maximum thickness is 17cm , 21% of the chord length, which is a possible value of thickness for an aerofoil.

Figure 8 – Typical velocity distribution in this investigation.



B Interesting display of the data which offers a visual representation of the data presented in the table.

B The use of arrows needs to be explained.

Compared to the real velocity distribution, the velocities I have found go down much too abruptly in the middle of the aerofoil, while they seem quite realistic on the extremes. This means that the method I found of calculating the distances travelled does not work very well for the middle parts of the aerofoil, but perhaps better on the tip and the tail. Although it is important to note that the graphs are for two different airfoils, and that on the first one an angle of attack was used, increasing the velocity on the upper side of the aerofoil, whereas I did not take this into account.

If we change the points on the graph so that they follow the trend of the real velocity distribution, we can find by how much the error in the velocity distribution has affected the findings. Roughly, the real velocity distribution is a straight line, so we can use arithmetic series to find a reasonable estimate of the average velocity, had the velocity distribution been right:

B The values used below correspond to those for a thickness factor h equal to 1.32 (see table p.8).

$$u_1 = 11.44 \quad u_8 = 9.53 \qquad S_n = \frac{n}{2}(u_1 + u_n)$$

$$S_8 = 4(11.44 + 9.53) = 83.88 \quad \text{so the average velocity is}$$

$$\frac{S_8}{8} = 10.49ms^{-1}$$

This velocity would give a lift force of roughly 182.36N, using the same equation as previously, and using the total planform area. The lift force I calculated for a thickness of 17cm was 86.84N. This is over 2 times less. This shows that the problems in the velocity distribution have led to a large underestimation of the lift force provided by the aerofoil. Was the velocity distribution more realistic, the relationship I have found should be translated horizontally by about 10 units and horizontally stretched by a factor of 2 so that it is much steeper. An estimated revision of my result could be approximately

B It is important to mention that this function is an estimation as more calculations are necessary to prove it.

Estimated Lift Force function $L_e(x) = 5.59 \times e^{0.30x} + 10$ where x is the thickness in cm.

Moreover, an important point is that this method can only work with an asymmetrical aerofoil, otherwise the distances travelled above and under would be the same and I would not be able to find a velocity differential. Then, this would have to do with the angle of attack, which falls outside of the scope of this investigation. Also, this investigation was purely in 2 dimensions that would underestimate the lift force, whereas more sophisticated and accurate models would be in 3 dimensions. Lastly, plane would be travelling at high speed before take-off and during the flight, so in fact, the speed of the wind would be much larger than 8m.s^{-1} on the lower side, and so would the lift force.

D *Critical analysis of the initial assumptions/ simplifications made for the investigation.*

Overall, I thought the relationship found through this method is reasonable, although not accurate, due to the problems with the theory, which underestimated the velocity of the wind on the upper side to a large extent. The investigation could be much improved by calculating the drag force, which is the limiting factor of the lift force. From these calculations, one could find the optimum thickness for an aerofoil of that type. Finally, this investigation is focused on the theoretical part, trying to predict what would happen as the thickness of an aerofoil increases. Hence, empirical evidence would be of great help to gain more precise information on the nature of the ULM aerofoil I investigated. This investigation has made me eager to deepen my understanding of fluid dynamics at university, and perhaps research into this topic further, to calculate a much more precise and accurate relationship between the thickness of an aerofoil and the lift force generated.

B, D *Interesting suggestion for possible and realistic improvements.*

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