

HORSE JUMPING

Jumping trainers always say that a rider needs to look for and strive towards a perfect take-off distance before a jump in order for the horse to clear the jump, to prevent the horse from getting tired and avoid accidents. The take-off is also the most important and the hardest part of the jump, since after it neither the horse nor the rider have any control over the jump. However, at the end of the day, even if you do not jump from the most perfect distance, the important thing is not to panic and ride as if nothing was wrong. That way you do not disturb the horse's rhythm and you give him the much needed confidence to jump over even the most difficult jumps. But of course there is only so much a horse can do, meaning that some jumps no matter how confident you are, are bound to go wrong. Since I am a rider myself, I wanted to investigate how much space before the take-off I can leave up to chance and what are the limiting distances from the jump above which the horse will not be able to complete the jump anymore. With my investigation I am hoping to interpret and support my practical riding knowledge with solid, more scientific mathematical theory and hopefully come to some conclusions that could serve me well while jumping in the future.



Picture 1



Picture 2

SOURCE OF PICTURE 1: *Equestrian and horse*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.equestrianandhorse.com/equipment/show-jumps.html>

SOURCE OF PICTURE 2: Grigg, K. *What I Know about Horseback Riding From Interior Design (Part 2): How They are Alike!*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.knottinghillinteriors.com/blog/2011/05/what-i-know-about-horseback-riding-from-interior-design-part-2-how-they-are-alike/>

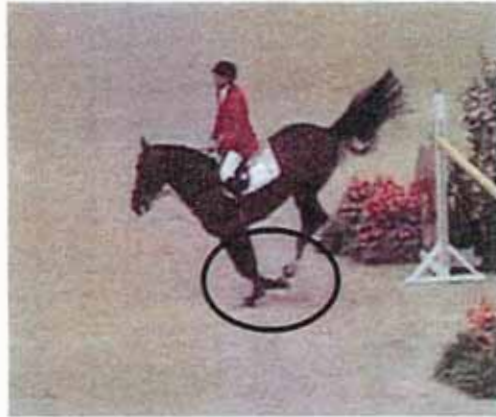
I decided to start my investigation by first finding the curve the horse makes when jumping over a jump and try to determine some of its characteristics. That will later help me to determine the limit values easier. According to many equestrian websites the trajectory the horse follows is parabolic, which I decided to test using images from different competitions and then breaking them down to smaller pieces.

One of the first things I had to decide on before observing the trajectory was the point on the horse I will be observing. Since the crucial points during the jump are the horse's legs, I focused on those and looking through many videos of people jumping I found out that it

does not matter whether I look at fore or hind legs. Especially when it comes to higher jumps, the points where all four legs leave the ground before the jump and the points where the four legs meet the ground again after the jump are approximately the same as it can be seen from the pictures below.



Picture 3



Picture 4

SOURCE OF PICTURES 3, 4: CentaurBiomechanics. 2012. *London 2012 Jumping Horses*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.youtube.com/watch?v=FzuZxnFLuQ>

Now that I have determined the points of observation, I can focus on the trajectory. The method I used for determining the horse's path during the jump was observation of many videos from jumping shows, which I then later put in slow motion and after that stopped the video in three most important moments of the jump – take-off, highest point and landing. This gave me a series of three pictures, for example:



Take-off

Picture 5



Peak of the jump

Picture 6



Landing

Picture 7

SOURCE OF PICTURES 5, 6, 7: Free Riding NZ- Alycia Burton. 2012. "Free Riding" *Jumping huge bareback*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.youtube.com/watch?v=0jBjg717TjC>

After that I linked the three moments together and put them in the same picture. By using a computer programme for drawing I drew the coordinate axes onto the picture, where the x-axis is at the level of the ground and the y-axis coincides with the jump (where there are more jumps one after another the y-axis coincides with the last one, which is usually also the highest one). This is what I got:



Picture 8 – SOURCE OF THE ORIGINAL PICTURE: Free Riding NZ- Alycia Burton. 2012. "Free Riding" *Jumping huge bareback*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.youtube.com/watch?v=0jBjg717TjC>

I repeated the same procedure in many different situations. I also found this picture that perfectly shows the trajectory of a horse during the jump:



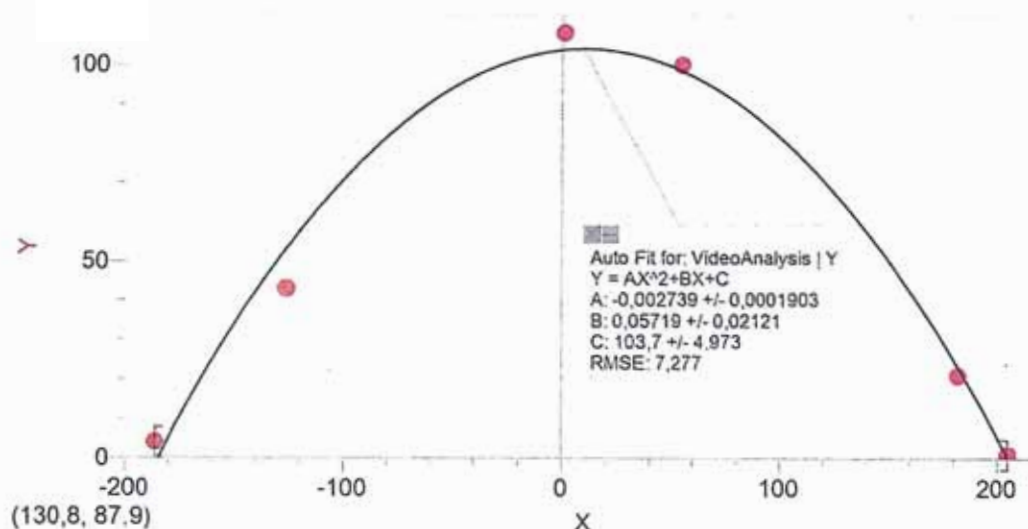
Picture 9 – SOURCE: *Horse jumping*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.flickr.com/photos/hooftprintsphotography/2538308731/sizes/z/in/photostream/>

To verify my observations I decided for an additional method of finding the horse's trajectory, namely by the use of Logger Pro programme, which allowed me to find the best approximation of the trajectory. I used the upper picture, imported it into the programme, plotted all the points where the horses legs are visible and then looked for the function that best fits the plotted points. This is what I got:



Picture 10 – SOURCE OF THE ORIGINAL PICTURE: *Horse jumping*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.flickr.com/photos/hooftprintsphotography/2538308731/sizes/z/in/photostream/>

As you can see the picture is not ideal, since it was taken at a specific angle and that is why I did not take the end of the highest jump as the y-axis, but the middle of that jump. Also the horse's legs when taking off and landing do not seem to be on the same line as the jump, which is why I put the x-axis a little bit higher than the ground is and consequently I also plotted the highest point of the jump a bit higher. However, this is not important in my case since, at this point of my investigation, I am only interested in the trajectory of the horse and not the distance from the jump. To see a better and clearer graph I decided to remove the original picture and keep only the plotted points, the coordinate axes and the best-fit curve. This is the graph I got:



Graph 1

It is understandable that not all points fit perfectly on the graph, since I plotted them by hand and we also must not forget that each horse has his own natural movement, meaning that if we observe a perfect parabolic trajectory on one horse there is very little chance we will observe the exact same trajectory on another horse. The reason the vertex of the graph does not lie on the y-axis, is again due to a specific angle the picture was taken at. Overall, I think that this graph strongly indicates that it does represent a quadratic equation, though not all points fit the graph perfectly. However, there is another thing that this graph shows very nicely and that is that the highest point up to which the horse gets during the jump, is a little bit higher than the jump itself. This point is therefore positioned above the vertex of the graph (as shown above), which is something that does happen in real life, since the horse needs to get over the jump and not fly into it.

As it can be seen from the examples above, the trajectory the horse makes while jumping really looks like parabolic or at least it is very close to being parabolic. There are some characteristics that can be observed from the graphs, namely the parabola is concave-down, which means that there is a negative sign before the x^2 in the function which can be introduced as $f(x)=ax^2+bx+c$, where a , b and c are real parameters, with a being different from 0 ($a \neq 0$) and x is the unknown variable. The vertex or the highest point on the graph is

the same as the height of the jump (or the height of the highest jump in the series). There is also another characteristic that can be observed, namely that the y-axis is the line of symmetry on my graphs, which makes the x-intercepts of the graphs equally apart from the jump. This indicates that the horse needs equal amount of space to take-off and to land.

Now, I will focus on the real parameters a , b and c , which are crucial for determination of the limiting distances from the jump at take-off.

In further explaining, letter k will denote the vertical translation up or down and letter h will denote horizontal translation left or right. So, if we take into consideration the quadratic function $g(x)=ax^2$ and translate it h units to the left or to the right and k units up or down, we can write the following function $f(x)=a(x-h)^2+k$, where a , k and h are real parameters and x is the unknown variable. We can rearrange the equation to get $f(x)=a(x^2-2xh+h^2)+k$, which can be again rearranged into $f(x)=ax^2-2ahx+ah^2+k$. If we compare this function with $f(x)=ax^2+bx+c$, we can see that they are the same when some rules are applied, namely that $b=-2ah$ and $c=ah^2+k$. This information will later help me to determine a , b and c parameters.

As I have mentioned before, I designed my coordinate system in such a way that y-axis is the axis of symmetry meaning that the graph is not translated left nor right, therefore $h=0$. It also means that the height of the jump, which also represents the y-intercept is equal to k (therefore the previously denoted y-axis will from now on be denoted as k-axis).



Picture 11 – SOURCE OF THE ORIGINAL PICTURE: Free Riding NZ- Alycia Burton, 2012. "Free Riding" Jumping huge bareback. [internet]. [cited 2. 12. 2013]. Available on: <http://www.youtube.com/watch?v=0jBjg717TJc>

From that and the fact that $b = -2ah$ we get the following:

$b = -2ah$; where I have previously determined that $h=0$, therefore b is also 0, meaning that there is no bx factor ($bx=0$) in the quadratic function $f(x)=ax^2+bx+c$ and that means that it now has the form of $f(x)=ax^2+c$.

When it comes to parameter c , I took into consideration the following equation:

$c=ah^2+k$; where again $h=0$, therefore $c=0+k=k$ and the quadratic function has now the form of $f(x)=ax^2+k$, where $k > 0$ and determined by the height of the jump.

As I have said before $a < 0$ and can be determined by x -intercept and height of the jump (k). According to the formula $x_1 \cdot x_2 = \frac{c}{a}$ where x_1 and x_2 represent the x -intercepts on the graph (and the distance between the take-off point and the jump in real-life situation) and $c=k$ as mentioned before, we get the following:

$x_1 \cdot x_2 = \frac{k}{a}$; where both x values differ only in sign, but not in magnitude, therefore, $x_2 = -x_1$ meaning that $x_1 \cdot x_2 = x_1 \cdot (-x_1) = -(x_1)^2$ and from that we get that that $-(x_1)^2 = \frac{k}{a}$ where $a < 0$ and $a \neq 0$. We can rearrange this equation into $(x_1)^2 = -\frac{k}{a}$ and since $a < 0$ we can therefore say that $\frac{k}{a} > 0$. This gives us $(x_1)^2 = \frac{k}{a}$ and that leads to $x_1 = \pm \sqrt{\frac{k}{a}}$.

All good and great, except for the fact that the function $p(x_1) = \pm \sqrt{\frac{k}{a}}$ has no upper limit value and x_1 can therefore take any real value except for 0; meaning that this mathematical model implies that a horse can jump over a jump at no matter what distance, even from 1km away, if necessary. This is of course unrealistic, which indicates that there is something wrong with the model I proposed.

Or maybe not all wrong completely, my explanation just lacks one final component, which is the angle at which the horse takes off. The angle I am referring to has its vertex at the take-off point (x_1), lower leg at the level of the ground and upper leg is made up of the horse's legs (I chose a line that represents the approximate position of his legs if both, left and right, were always at the same position). On the picture the angle would look something like this:



Picture 12 – SOURCE OF THE ORIGINAL PICTURE: Free Riding NZ- Alycia Burton. 2012. "Free Riding" Jumping huge bareback. [Internet]. [cited 2. 12. 2013]. Available on: <http://www.youtube.com/watch?v=0jBjg717Tjc>

This angle (α) is getting smaller and smaller as the distance from the jump increases, therefore I need to determine the maximum and minimum value of the angle, which is definitely somewhere between 0° and 90° . The upper limit (90°), which can be observed when the horse takes-off close to the jump, does not pose a real problem, since it can happen that a horse when jumping extremely big jumps (I am talking about 1,80m or higher jumps) takes-off at an angle near 90° . The problem is the lower limit, which we can see when the horse takes-off further away from the jump, since if it is too small, the distance calculated from the jump will again be too large. Therefore I have decided for the purposes of this exploration to limit the angle to the smallest possible value of 30° . For the value of k (the height of the jump) I took the most common 100cm jump. What I had to do next, was to link together the angle at which the horse takes off (α) with the slope of the function that makes up the horse's jumping trajectory and then find the x-intercept or the take-off point.

I started by finding the first derivative of the function $f(x)=ax^2+k$, which was:

$f'(x)=2ax$, where $a<0$ and x is the unknown variable, which means that the straight line is a decreasing one. If we try and represent the equation of a straight line $y=2ax$ in an explicit form (which is $y=mx+n$, where m and n are real parameters, m being the slope of the line and n being the y-intercept the line makes with the y-axis), we get $y=2ax+n$, with $2a=m$. To determine n we need to remember that this line ($y=2ax+n$) needs to go through point k on the y-axis, where $k=100$, therefore we can say that $n=k=100$, getting the equation of the line $y=2ax+100$. The thing we need to remember here is that I have previously mentioned that the highest point up to which the horse gets during the jump, is a little bit higher than the jump itself, meaning that n is actually a bit more than 100. However since this additional value is significantly smaller than 100 and different for every horse, I decided to omit it for the purposes of my investigation.

By doing this I am now able to find values that correspond to the angle of α , for which $30^\circ < \alpha < 90^\circ$ and by finding those values, I can then calculate x values by using the formula $y=2ax + 100$, where $y=0$, since we are looking for the x-intercept.

I used the formula which links together the slope of the straight line and the angle (α) that line makes with the x-axis:

$\tan\alpha = \text{slope}$ and in my case the slope is $2a$, therefore $\tan\alpha = 2a$. Now if I look at the limit values of the angle that I chose:

$\alpha=30^\circ$, which leads to $a = \frac{\tan \alpha}{2} = \frac{\tan 30^\circ}{2} = 0,2887$ (rounded to 4 significant figures for better accuracy) this then gives $0=2 \cdot 0,2887x+100$, therefore $x= -173,19$. The negative sign comes from the fact that the angle between the slope and the positive side of the x-axis is actually between 90° and 180° , but since 30° and 150° are supplementary angles their tan values differ only in sign, but not in magnitude. So, $\tan 150^\circ$ takes a negative value which makes a negative and that makes x positive, meaning that $x=173,19\text{cm}$.

For the upper value I took the angle $\alpha = 85^\circ$, since $\tan\alpha = 90^\circ$ does not exist and reasonably speaking the horse never actually takes off at the angle of exactly 90° .

$\alpha=85^\circ$, which leads to $a = \frac{\tan \alpha}{2} = \frac{\tan 85^\circ}{2} = 5,7150$; which gives $0=2 \cdot 5,7150x+100$ therefore $x=-8,7489$; but again for the same reason I have mentioned above I take the positive value,

that is $x=8,75\text{cm}$, which is quite a small value. However, that is understandable since I took such a large angle.

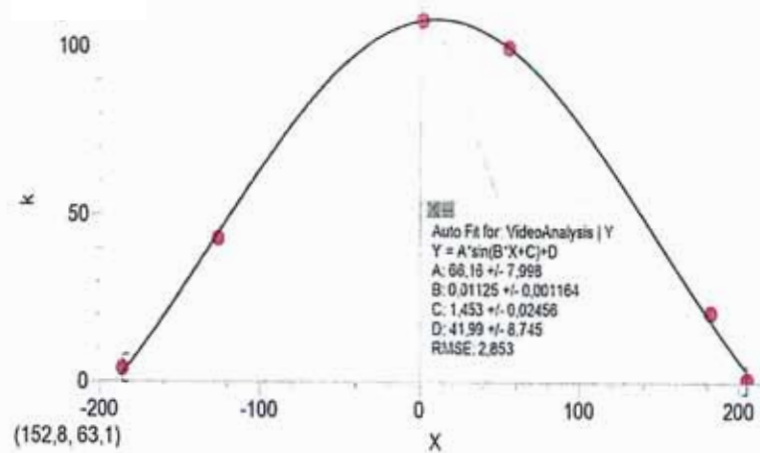
The results of my investigation indicate that there is quite a lot free space before take-off ($173,19\text{cm}-8,75\text{cm}=164,44\text{cm}=1,64\text{m}$). However, if we take into consideration that a horse has in average 3-meters-long gallop stride, 1,64m of free-space before the jump does not seem such a large number anymore. What is more, it is exactly this free space that usually causes problems to the rider and the horse, since it can happen that the horse comes up to

the jump at such a distance that the take-off from 1,64m away is too demanding for him, but if he takes another gallop stride, he will knock the jump down. This can to some extent be avoided with practice and by knowing the abilities of your horse, but accidents do happen anyway. There are some things we need to be careful about when it comes to jumping, namely that we do not get our horse too tired before every jump, which is exactly what happens if we jump from very big or very small distances.

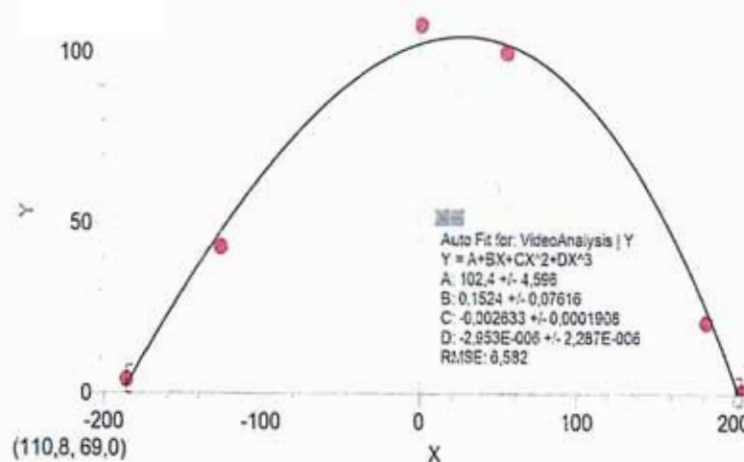
Concerning my investigation, there are quite a lot of opportunities for error in my calculations, since there are some things that I have estimated on my own from my riding knowledge, for example the angles. There are also some things that I have determined by observing video footage of people riding, such as that the trajectory looks like parabolic and the characteristics of the parabola. Both of these methods were in my opinion necessary for this investigation, however I am aware that they were probably the source of some of the mistakes, the major one being that I cannot be sure whether the trajectory really is parabolic or is it something else. To show that there are many different possibilities for the horse's trajectory, depending on how the horse jumps and how you plot the points onto your picture, I decided to try and find a different curve of the trajectory by using Logger Pro. I decided to plot a cubic and a sine curve to see whether they are more appropriate than a parabola and this is what I got:



Picture 13 – SOURCE OF THE ORIGINAL PICTURE: *Horse Jumping*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.flickr.com/photos/hooftprintsphotography/2538308731/sizes/z/in/photostream/>



Graph 2 - a sine curve



Graph 3 - a cubic curve

From the graphs above we can see that a sine curve fits the plotted points the best. However, I believe that even though I based my research on a parabolic trajectory it is still relevant, since the fact that I plotted the point onto the picture by hand leaves quite a lot of room for error.

Also the fact that I based my research on a small sample of jumps contributes a lot to mistakes, which can be seen from the example below. The picture shows a horse jump, where the rider (or the horse) decided to take off far away from the jump, even though they could probably come another gallop stride closer.



Picture 14 – SOURCE OF THE ORIGINAL PICTURE: *Horse jumping*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.flickr.com/photos/colonelkillgore/3089645047/sizes/z/ir/photostream/>

Even without any use of technology it is evident that the take-off and landing point are not equally distanced from the jump as I have determined for the purposes of my research. This indicates that the model I proposed does not work in all situations. There is also a question whether the points where all four legs of the horse leave the ground before the jump and the points where the four legs meet the ground again after the jump really are the same as I have estimated at the beginning of my research.

Though I believe my investigation to be quite successful, it has one major drawback and that is that it is a theory. In practise all of the information I gathered in this paper become irrelevant, there is just you, the horse and a huge jump and not a lot of people have time to think about the point of take-off. However, this kind of research might be useful for professional riders who look into every detail of the jump, walk through every meter of the jumping course and are consequently rewarded with brilliant results. Those kinds of jumpers are trained to spot the perfect distance from very far away and that is one of the most important factors separating professionals from amateurs. But sometimes also they get it wrong and the level of their mistake determines whether they will have to say goodbye to a gold medal,



Picture 15 – SOURCE: Faulkner, K. *The moment Zara's gold dream ended: Royal's horror as her horse clips fence during show-jumping final.* [internet]. [cited 2. 12. 2013]. Available on: <http://www.dailymail.co.uk/news/article-2181431/London-2012-Olympics-Medal-Team-GB-Zara-Phillips-eventing-team-ride-glory.html>

or maybe even to their careers.



Picture 16 – SOURCE: *Horse ride or horse fight?.* [internet]. [cited 2. 12. 2013]. Available on: <http://english.peopledaily.com.cn/90002/94411/94445/6478777.html>

I also have to mention some external factors determining the jump, such as the ground riders are competing on, the confidence of the rider and the horse etc. We also need to consider the horses strength (and the riders for that matter), since not all horses are able to make a jump from a very big or very small distance. However, most of those disturbances can be nicely avoided with practise and it is practise that helps you find the perfect jump-off.

SOURCES

CentaurBiomechanics. 2012. *London 2012 Jumping Horses*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.youtube.com/watch?v=FzuZxnFLuQI>

Equestrian and horse. [internet]. [cited 2. 12. 2013]. Available on: <http://www.equestrianandhorse.com/equipment/show-jumps.html>

Faulkner, K. *The moment Zara's gold dream ended: Royal's horror as her horse clips fence during show-jumping final*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.dailymail.co.uk/news/article-2181431/London-2012-Olympics-Medal-Team-GB-Zara-Phillips-eventing-team-ride-glory.html>

Free Riding NZ- Alycia Burton. 2012. *"Free Riding" Jumping huge bareback*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.youtube.com/watch?v=0jBjg717TJc>

Grigg, K. *What I Know about Horseback Riding From Interior Design (Part 2): How They are Alike!*. [internet]. [cited 2. 12. 2013]. Available on: <http://www.knottinghillinteriors.com/blog/2011/05/what-i-know-about-horseback-riding-from-interior-design-part-2-how-they-are-alike/>

Horse jumping. [internet]. [cited 2. 12. 2013]. Available on: <http://www.flickr.com/photos/hoofprintsphotography/2538308731/sizes/z/in/photostream/>

Horse jumping. [internet]. [cited 2. 12. 2013]. Available on: <http://www.flickr.com/photos/colonelkillgore/3089645047/sizes/z/in/photostream/>

Horse ride or horse fight?. [internet]. [cited 2. 12. 2013]. Available on: <http://english.peopledaily.com.cn/90002/94411/94445/6478777.html>

Jump in the horse. [internet]. [cited 2. 12. 2013]. Available on: <http://bowlingsite.mcf.com/movement/hjump.html>