Part 1: Introduction

In the IB Mathematics Higher Level syllabus, I learnt probability and statistics as core topics. This technique is widely applicable in our daily lives as we always take risks and expose ourselves to a variety of choices where decision making is crucial. We are often carrying out trials which has a chance of success but it is not definite. This is a concept of probability that provides us with the understanding of the different choices and helps us take control of them.

In Mathematics, we learn the idea to adopt mathematical reasoning to uncertainties, especially when we should not solely reply on our intuition to make judgements. We often make use of probability to decide on a conclusion that gives the best outcome. One of the most frequent uses of probability is definitely in the field of betting (such as gambling). For example, theoretically, the probability of throwing a fair dice with "1" as the result is $\frac{1}{6}$. However, even if the dice is thrown for 12 times, the number of times "1" shows up may not be $12\times\frac{1}{6}$. It is concluded that the number of times the experiment conducted is not large enough to justify the claim, this leads to the question "How many times should it be tested in order to attain an accurate and precise probability?"

Therefore, in this paper, I am going to simulate a probability game - The Monty Hall problem - using Geogebra. This topic is inspired by a movie "21", in which the professor makes use of this paradox to introduce conditional probability. Through analyzing results from the probability simulation, the potability of winning between the two choices can be derived. After that, I would make use of the Bayesian Rule of Probability and Law of Total Probability to prove the results obtained to draw comparisons between the experimental values and the theoretical values. Finally, I would extend the exploration by investigating on the different variations of the Classic Monty Hall Problem. Ultimately, the generalized winning strategy can be acquired.

Part 2: Overview of Game theory and Bayesian Rule of Probability

Game theory is "the study of mathematical models of conflict and cooperation between intelligent rational decision-makers." (Myerson, 1991) This is particularly applicable in games (such as gambling, card games and chess) as well as real-world problems varied from economics, to politics and warfare. (Wolfram, 2014)

"Probability is a branch of mathematics that deals with calculating the likelihood of a given event's occurrence." (Rouse, 2005) Probability is expressed in terms of a numerical measure with the values between 0 and 1, where 0 indicates impossibility and 1 indicates certainty. (Feller, 1968) There are different types of probability, and in Monty Hall, a specific type of probability is applied - conditional probability. Conditional probability is applicable when the "probability of some event A, given the occurrence of some other event B. Which can be expressed as $P(A\mid B)$, where,

$$P(A\mid B)=\frac{P(A\cap B)}{P(B)}$$

Event A and B can be expressed as P(A∩B), which is the probability of even A, P(A), times the probability of event B, P(B), if A and B are independent.

Part 2.1: Bayesian Rule of Probability and Law of Total Probability

As mentioned, the conditional probability of events A and B is given by

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

By rearranging the conditional probability, we have

$$P(A \cap B) = P(A \mid B) \times P(B) \qquad \textbf{[2.1.1]}$$

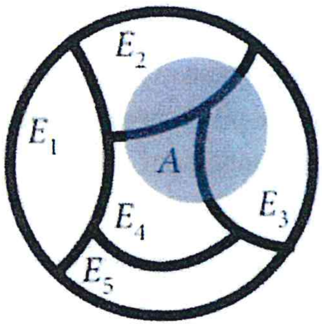
Then, by interchanging A and B, we have

$$P(B \cap A) = P(B \mid A) \times P(A)$$

Since P(A∩B) = P(B∩A), the Bayesian Rule of Probability shows

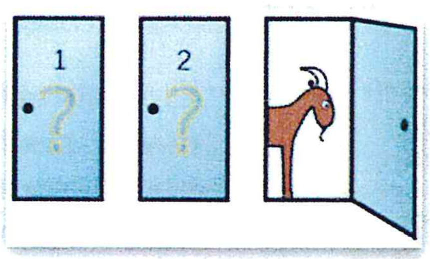
$$P(A \mid B) = \frac{P(A)P(B \mid A)}{P(B)} \qquad \textbf{[2.1.2]}$$

The Law of Total Probability illustrates the idea that, in general, the probability of occurrence of event A is an outcome of several events. Consider the diagram below,



The probability of A can be given by $\Pr(A) = \Pr(A \cap E_1) + (\Pr(A \cap E_2) + \ldots + \Pr(A \cap E_5)$.

Part 3: The Monty Hall problem

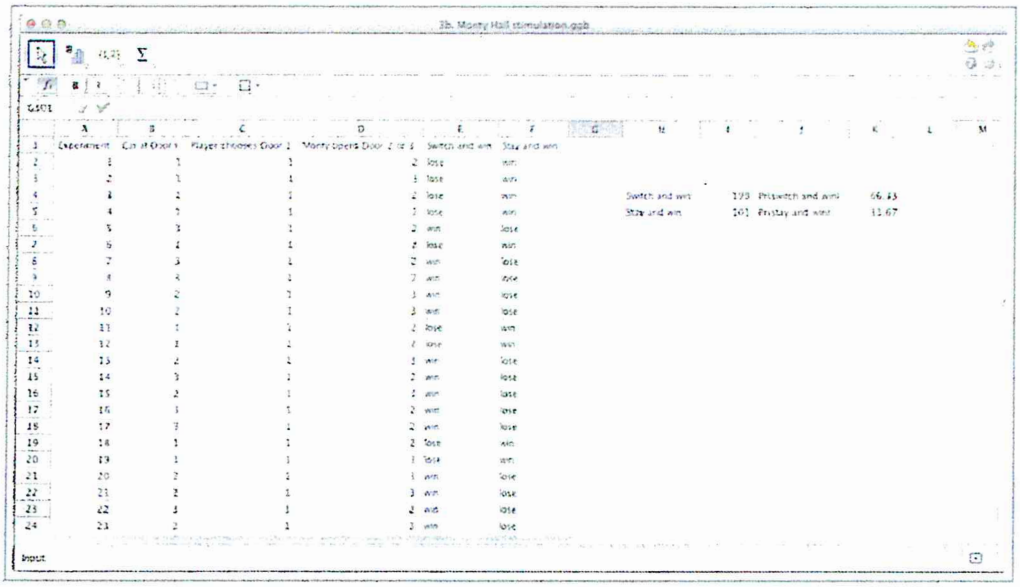


The Classic Monty Hall problem originated from the game show "Let's Make a Deal", presents a scenario where there are three doors, with goats behind two and a car behind the remaining door. The host - Monty - will allow the player to pick one door, and Monty will open one (which conceals a goat) of the two remaining doors. Lastly, Monty offers the player a chance to switch his or her decision.

Thus, the question is, "*Should the player switch to another door or stick to his original choice*?"

Part 3.1: Probability simulation of Classic Monty Hall problem

A probability simulation is often used to model random events in order to simulate the outcomes of real life situations. In this investigation, the software Geogebra is used to create a probability simulation creating 300 trials of Monty Hall problem.



	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Experiment	Car at Door i	Player chooses Door 1	Monty opens Door 2 or 3	Switch and win	Stay and win							
2	1	1	1	2	lose	win							
3	2	1	1	3	lose	win							
4	3	1	1	2	lose	win		Switch and win		199	Pr(switch and win)	66.33	
5	4	1	1	3	lose	win		Stay and win		101	Pr(stay and win)	33.67	
6	5	3	1	2	win	lose							
7	6	1	1	2	lose	win							
8	7	3	1	2	win	lose							
9	8	3	1	2	win	lose							
10	9	2	1	3	win	lose							
11	10	2	1	3	win	lose							
12	11	1	1	2	lose	win							
13	12	1	1	2	lose	win							
14	13	2	1	3	win	lose							
15	14	3	1	2	win	lose							
16	15	2	1	3	win	lose							
17	16	3	1	2	win	lose							
18	17	3	1	2	win	lose							
19	18	1	1	2	lose	win							
20	19	1	1	3	lose	win							
21	20	2	1	3	win	lose							
22	21	2	1	3	win	lose							
23	22	3	1	2	win	lose							
24	23	2	1	3	win	lose							

Fig.1 - Overview of simulation consisting of 300 trials of Monty Hall situations

The simulation shows the different outcomes of possible combinations. Since the outcome depends on the position of the car, the choice of Monty as well as the final decision of the player, all of the above elements are included in the simulation.

	A	B	C	D	E	F
1	Experiment	Car at Door i	Player chooses Door 1	Monty opens Door 2 or 3	Switch and win	Stay and win
2	1	1	1	2	lose	win
3	2	1	1	3	lose	win
4	3	1	1	2	lose	win

In the second column (column B), it shows the location of car. There are three possible locations to conceal the car - Door 1, Door 2 and Door 3. Hence the location of car at Door i can be given by inputting a function "RANDBETWEEN[1,3].

B2 =RandomBetween[1, 3]

	A	B	
1	Experiment	Car at Door i	Player ch
2	1	1	

The function can generate random integers 1, 2 and 3 to simulate the probability that the car is behind Door 1, Door 2 or Door 3.

In the third column (column C), it indicates the player's initial choice, which is assumed to be Door 1 all the time to minimize the number of variables and strengthens the results.

C2 =1

	A	B	C
1	Experiment	Car at Door i	Player chooses Door 1
2	1	1	1

In the forth column (column D), it shows the door that Monty chooses to open after the player has picked his or her choice (which is always Door 1). Therefore, Monty can only choose to open Door 2 or Door 3. According to the game, Monty will only reveal the door that conceals a goat. Therefore, the function used is "IF[B2==1, RANDOMBETWEEN[2,3], IF[B2==2,3,2]]"

D2 =If[B2 - 1, RandomBetween[2, 3], If[B2 - 2, 3, 2]]

	A	B	C	D
1	Experiment	Car at Door i	Player chooses Door 1	Monty opens Door 2 or 3
2	1	1	1	2

This input function indicates that if B2 (location of car) equals to 1 (meaning that the car is behind Door 1), then Monty will randomly pick the remaining doors as both conceal a goat. However, if the car is hidden behind Door 2, then Monty can only choose Door 3 to open, or else Monty will open Door 2.

Lastly, the simulation calculates the probabilities of winning if the player switches doors or stick with his or her original decision.

E2 If[B2 ≠ C2, "win", "lose"]

	A	B	C	D	E
1	Experiment	Car at Door i	Player chooses Door 1	Monty opens Door 2 or 3	Switch and win
2	1	1	1	2	lose

F2 If[B2 - C2, "win", "lose"]

	A	B	C	D	E	F
1	Experiment	Car at Door i	Player chooses Door 1	Monty opens Door 2 or 3	Switch and win	Stay and win
2	1	1	1	2	lose	win

The function for "switch and win" is "IF[B2≠C2, "win", "lose"]". This means that if the player's initial choice is not the door that conceals the car, he or she will win if he or she switch decision (since Monty already eliminated the one with goat). Nevertheless, if the player's original decision (C2) is equals to the location of the car (B2), the player will win if he or she stays, hence the input function "IF[B2==C2, "win", "lose"]".

Part 3.1.1: Results obtained from probability simulation

Based on the simulation of 300 trials of the Classic Monty Hall problem, the number of win if the player chooses to switch is 199 games out of 300 games. Whereas, if the player sticks with initial decision, the number of win is 101 out of 100.

Switch and win:	199	Pr(switch and win):	66.33
Stay and win:	101	Pr(stay and win):	33.67

Part 3.2: Solution to Classic Monty Hall Problem

"Let it be! It does not matter." This may be the most common response to this problem, as most people claim that the probability of winning and loosing is the same since there are only two doors left. However, the answer is quite counter-intuitive in the sense that the player should always switch in order to increase his or her chance of winning.

Referring to the situation, after Monty reveals the door concealing one goat behind it, there are only two doors left - with a car and a goat behind the two doors respectively. To illustrate the scenario, consider the following table:

Door 1	**Door 2**	**Door 3**	**Switch and win**	**Stay and win**
Car	Goat	Goat	Goat	Car
Goat	Car	Goat	Car	Goat
Goat	Goat	Car	Car	Goat
(Door 1 is always the player's first choice)			$\text{P(Car)} = \frac{2}{3}$	$\text{P(Car)} = \frac{1}{3}$

Assuming the player always chooses Door 1 as his or her original choice, Monty can choose to open either Door 2 or Door 3. Suppose the car is behind Door 1 (first row), then Monty can open either Door 2 or Door 3. If the player switches his or her decision, he or she looses. However, if the player sticks with Door 1, he or her wins.

The second row illustrates the scenario where the car is behind Door 2, hence Monty can only open Door 3 where it conceals the goat. In this case, if the player switches his or her choice, he or she wins. The third row depicts a similar idea except the car is hidden behind Door 3.

Therefore the probability of winning is $\frac{2}{3}$ if the player switches his or her decision, whereas the probability of winning is only $\frac{1}{3}$ if the player sticks to his or her original choice.

At the beginning of the game, the chance of choosing a goat is higher, with the probability of $\frac{2}{3}$, while the probability of winning a car is only $\frac{1}{3}$. Hence, the player should always switch away his or her wrong choice in order to obtain a higher probability to win.

Part 3.3: Law of Total Probability in Classic Monty Hall Problem

This solution can be denoted by using the Law of Total Probability.

Let C_i be an event that the car is behind Door i,

$$\Pr(C_1) = \Pr(C_2) = \Pr(C_3) = \frac{1}{3} \qquad \textbf{[3.3.1]}$$

Then, let O_{ij} be a compound event that represents the player's initial choice Door i and Monty's decision of opening Door j. According to the game rule, Monty does not reveal the door of the player's choice, hence,

$$\Pr(O_{11}) = \Pr(O_{22}) = \Pr(O_{33}) = 0$$

Therefore in the game, given that the player chooses Door 1 (i = 1)as his or her initial choice, Monty then has to open Door 2 or Door 3 ($\Pr(O_{12})$ or $\Pr(O_{13})$), where the situation shows $i \neq j$. The probability of $\Pr(O_{12})$ can then be determined by using the Law of Probability.

By the Law of Total Probability,

$\Pr(O_{12}) = \Pr(O_{12} \cap C_1) + \Pr(O_{12} \cap C_2) + \Pr(O_{12} \cap C_3)$

$= \Pr(O_{12} \mid C_1) \times \Pr(C_1) + \Pr(O_{12} \mid C_2) \times \Pr(C_2) + \Pr(O_{12} \mid C_3) \times \Pr(C_3)$ [According to 2.1.1]

$= \frac{1}{2} \times \frac{1}{3} + 0 \times \frac{1}{3} + 1 \times \frac{1}{3} = \frac{1}{2}$

The above can be explained using conditional probability. According to (3.3.1), the probability of car placed behind Door 1, Door 2 and Door 3 is $\frac{1}{3}$ (i.e $\Pr(C_1)$, $\Pr(C_2)$ and $\Pr(C_3) = \frac{1}{3}$). If the player chooses Door 1 and the car is behind Door 2, Monty only has the choice to open Door 2 or Door 3. Hence the probability of Monty opening Door 2 after the player chooses Door 1, given that the car is actually behind Door 1 is $\frac{1}{2}$ (i.e $\Pr(O_{12} \mid C_1) = \frac{1}{2}$). Whereas, if the player chooses Door 2 and the car is behind Door 2, the probability of Monty opening Door 2 is 0 because he can only reveal a car concealing a goat according to the game. Similarly, the probability of Monty opening Door 3 and the player chooses Door 2 given that the car is behind Door 1, $\Pr(O_{23} \mid C_3)$, should be 1. Therefore, using the Law of Total Probability, the probability of the player choosing Door 1 and Monty revealing Door 2 is 0.5.

Eventually, conditional probability can be used to explain the Monty Hall problem. The problem is:

> *"Given that the player chooses Door 1, and Monty opens Door 2 or Door 3, what is the probability that the player can get the car by staying with his or her initial choice (Meaning that the car is actually behind Door 1)?"*

The theoretical value - $\frac{1}{3}$ - has already been determined in the above. Here, the Bayesian Rule of Probability can be used to derive the answer through conditional probability. Assuming Monty opens (eliminates) Door 2,

$$\Pr(C_1 \mid O_{12}) = \frac{\Pr(C_1)\Pr(O_{12} \mid C_1)}{\Pr(O_{12})}$$

$$= \frac{\left(\frac{1}{3}\right) \times \left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{1}{3}$$

As stated above, $\Pr(C_1) = \frac{1}{3}$ and $\Pr(O_{12} \mid C_1)$ illustrates the car is behind Door 1, the player chooses Door 1 and Monty opens Door 2 with probability of choosing between Door 2 and Door 3 by the host is $\frac{1}{2}$. Thus the result shows the probability is $\frac{1}{3}$, which matches the theoretical value above.

On the other side, it is claimed that: *"If the player makes a wrong choice at the beginning, his or her probability to win is $\frac{2}{3}$ by switching."*

Assuming that the car is now behind Door 3 and the player's initial choice is Door 1. In this case, since Monty cannot reveal the Door concealing a car, he only has the choice to open Door 2.

$$\Pr(C_3 \mid O_{12}) = \frac{\Pr(C_3)\Pr(O_{12} \mid C_3)}{\Pr(O_{12})}$$
$$= \frac{\left(\frac{1}{3}\right)\times(1)}{\left(\frac{1}{2}\right)} = \frac{2}{3}$$

The result matches the argument in the previous section.

Part 3.4: Comparing theoretical and experimental values

In general, the probability of win in the Classic Monty Hall game can be expressed as:

	Experimental	Theoretical
Switch and win	$= \frac{199}{300} \times 100\% = 66.33\% \approx 66.67\%$	$= \frac{2}{3} \approx 0.6667 \approx 66.67\%$
Stay and win	$= \frac{101}{300} \times 100\% = 33.67\% \approx 33.33\%$	$= \frac{1}{3} \approx 0.3333 \approx 33.33\%$

- Pr(switch and win): The experimental value is close to the theoretical value with only ±0.005% error.
- Pr(stay and win): The experiment value is also close to the theoretical value with the percentage error of ±0.01%.

From the two results obtained, it can be concluded that the experimental values match the theoretical values, indicating the accuracy of this probability simulation.

Part 4: Variations

After deciding that the probability winning if the player switches his or her decision is greater than staying with the initial choice in the Classic Monty Hall problem, would different variations of the game affect the probability of winning?

Part 4.1: Monty does not know the location of the car

In this version, there are three identical doors, with a car behind one and goats behind the remaining two. However, Monty does not have the information of where the car is, hence he chooses a door randomly after the player has chosen his or her decision.

In the classical version, the probability of the player guessing correctly in his or her first choice is $\frac{1}{3}$. Meaning that the probability of the initial guess being incorrect is $\frac{2}{3}$. Hence, the probability of winning is greater if the player chooses to switch his or her choice.

However in the case where Monty does not know the location of the car and the player's initial choice was incorrect, the probability of Monty opening the door with a car is $\frac{1}{2}$. This ends the game immediately. The probability that Monty ends the game by choosing the door with a car randomly is $\frac{1}{2}\times\frac{2}{3}=\frac{1}{3}$.

Therefore, the probability of winning in this case decreases as Monty uses it on his random decisions.

Part 4.2: Car not placed randomly

In all previous variations, only the number of doors or behaviour of Monty have changed but the car was placed randomly in all cases. Would the probability of car being placed in different doors affect the final outcome of the game?

"Assuming that the car is not placed randomly behind the three doors. Instead, with the probability of the car behind Door 1 as P_i, the probability behind Door 2 as P_2 and the probability behind Door 3 as P_3 (in which $P_1 \geq P_2 \geq P_3$), and Monty always opens a door that conceals a goat. Suppose the player chooses Door 1 as his or her initial choice and Monty reveals the weighted probability of the doors to the player, should the player switch doors or stay with his or her original decision?"

For the classic Monty Hall version, the probability of car behind the three doors are $p_1 = p_2 = p_3 = \frac{1}{3}$. However, in this variation, the probability of car behind the doors are $p_1 + p_2 + p_3 = 1$. For example, the probabilities can be $P_1 = \frac{4}{8}$, $P_2 = \frac{3}{8}$ and $P_3 = \frac{1}{8}$, corresponding to $p_1 > p_2 > p_3$.

Firstly, denote Ci as the event that the car is behind Door i. Thus, the probability can be expressed as,

$$\Pr(C_1) = p_1 = \frac{4}{8}, \Pr(C_2) = p_2 = \frac{3}{8}, \Pr(C_3) = p_3 = \frac{1}{8}$$

Also, denote O_{ij} as the event that Monty opens Door j after the player picked his or her choice. Since Monty never opens the door with the car, it is always

$$\Pr(O_1 \mid C_1) = \Pr(O_2 \mid C_2) = \Pr(O_3 \mid C_3) = 0$$

According to the Bayesian Rule of Probability (according to 2.1.2) , the above probability can be expressed as

$$\Pr(C_1 \mid O_2) = \frac{\Pr(C_1)\Pr(O_2 \mid C_1)}{\Pr(O_2)}$$

in which,

- $\Pr(C_1) = p_1 = \dfrac{4}{8}$
- $\Pr(M_2 \mid C_1) = \dfrac{1}{2}$ → (the probability of the event that Monty opens Door 2 out of the 2 choices (Door 2 and Door 3) given that the car is in Door 1)

The Law of Total Probability can now be used to calculate $\Pr(M_2)$.

$\Pr(M_2) = \Pr(C_1)\Pr(M_2 \mid C_1) + (\Pr(C_1')\Pr(M_2 \mid C_1')$ Where C_1' is the complementary event of C_1

$= \frac{4}{8} \times \frac{1}{2} + (1 - \frac{4}{8}) \times \frac{1}{2}$

Hence, substituting the above values into $\Pr(C_1 \mid O_2)$, the value is

$$\Pr(C_1 \mid O_2) = \frac{\left(\frac{4}{8}\right) \times \left(\frac{1}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{4}{8} = p_1$$

Therefore, assuming that the player has chosen Door i as his or her initial choice, the probability of the car behind Door i after Monty opened one door and revealed the weighted doors is still P_i. The door Monty opened will then have a probability of 0. This means that the probability of the car behind other remaining door will be $1 - P_i$.

In conclusion, switching is only preferable if $1\text{-}P_i > P_i$. To maximize the probability of winning, the player should choose the door with the smallest P_i, so by switching, the player can obtain $1\text{-}P_i > P_i$. In this case where $P_1 \geq P_2 \geq P_3$, the player initially chose Door 1 which has the greatest P_i, hence switching is not favorable and the player should stick to his or her initial door in order to increase his or her probability to win. Ultimately, the winning strategy is dependent on the probabilities of the three weighted doors and if the player initially chose the door with the greatest probability, he or she should stay; whereas if the player chose the door with the least probability to win, he or she should switch.

Part 4.3: *n*-doors and Monty opens *n-2* doors (leaving 2 doors available)

After exploring the different variations in the classic Monty Hall version, I wish to further extend the investigation by introducing n doors into the game. In this version, there are n doors. Monty asks the player to choose *one* door and Monty would open the other n-2 doors concealing goats. At the end, there are 2 doors remaining, one with the car and one with the goat. In this case, should the player still switch his or her decision?

In theory, switching between many choices increases the chance to win. For example, there are 4 doors and Monty opens 2 doors. Assuming that the player chose the incorrect door as his or her initial decision, which has the probability of $\frac{3}{4}$. If the player switches his or her decision after Monty opens the 2 doors concealing goats, the probability of winning switches from $\frac{1}{4}$ to $\frac{3}{4}$. Thus, the more doors there are, probability of wining by switching away the initial decision increases.

This can be summarizes in the below table:

n-doors	Pr(Car)	Pr(Goat)	Pr(Switch and win)
4	1/4	3/4	3/4
5	1/5	4/5	4/5
6	1/6	5/6	5/6
7	1/7	6/7	6/7
n	$\frac{1}{n}$	$\frac{n-1}{n}$	$\frac{n-1}{n}$

As shown in the table above, the probability of the player winning by switching his or her initial decision with n-doors is $\frac{(n-1)}{n}$. This further enhances the effect of switching on the probability of winning.

Part 4.4: n-doors and Monty opens *1* door (leaving n-1 doors available)

Similar to Part 4.4, this variation also has n doors where they conceals 1 car and n-1 goats. However, this time Monty chooses not to open n-2 doors, but only 1 of the doors aside from the player's initial choice. This leaves n-2 doors available for the player to switch into. Does this increase the probability of winning?

Consider the following comparison:

Stay	Switch
$\Pr(win) = \frac{1}{n}$	$\Pr(win)$ = Pr(a goat behind first door) × Pr(a car behind the second door given that a goat behind the first door) $= \frac{(n-1)}{n} \times \frac{1}{n-2}$ $= \frac{n-1}{n-2} \times \frac{1}{n} > \frac{1}{n}$

The table shows the probability of winning between the decisions of staying with initial choice and switching to another door. In the left column, it shows the probability of winning if the player chooses to stick with his or her original choice, which is $\frac{1}{n}$ (one car out of n doors). On the other hand, the right column shows the probability of winning if the player switches doors. Here, conditional probability is used to derive the answer. As shown, the probability of switching and win is $\frac{n-1}{n-2} \times \frac{1}{n}$, in which $\frac{n-1}{n-2}$ is positive. As $\frac{n-1}{n-2} \times \frac{1}{n}$ is greater than $\frac{1}{n}$, therefore the probability of winning by switching away the player's initial choice is greater than not switching in this case.

Part 5: Conclusion: Generalized winning strategy

In general, the table below summarizes the winning strategy for the above variations.

1. Classic Monty Hall (Part 3.2)

- 3 Doors
- Player chooses 1 door and Monty opens 1 door
- 1 Car (car placed randomly)
- Monty knows the location of the car
- 2 Goats
- Monty is honest

Pr(car)	$\frac{1}{3}$
Pr(goat)	$\frac{2}{3}$
Pr(stay and win)	$\frac{1}{3}$
Pr(switch and win)	$\frac{2}{3}$
Winning strategy	Switching is more favorable.

2. "Ignorant Monty" (Rosenthal, 2005) (Part 4.1)

- 3 Doors
- Player chooses 1 door and Monty opens 1 door
- 1 Car (car placed randomly)
- **Monty does not know the location of the car**
- 2 Goats
- Monty is honest

Pr(car)	$\frac{1}{3}$
Pr(goat)	$\frac{2}{3}$
Pr(stay and win)	$\frac{1}{3}$
Pr(switch and win)	$\frac{2}{3}$
Winning strategy	Probabilities of winning by both switching and sticking to initial decision are the same as the Classic Monty Hall version.

3. Weighted Doors (Part 4.2)

- 3 Doors
- Player chooses 1 door and Monty opens 1 door
- **1 Car (car not placed randomly)**
- Monty knows the location of the car
- 2 Goats
- Monty is honest

Pr(car)	$P_1 //\ \ 2$
Pr(goat)	$1 - (P_2$
Pr(stay and win)	$P_1 //\ \ 2$
Pr(switch and win)	$1 - (P_2$
Winning strategy	Depends on the different probabilities of the three doors and the initial decision made by the player.

4. *n*-Doors (Part 4.3)

- ***n* Doors**
- Player chooses 1 door and **Monty opens *n*-2 doors**
- 1 Car (car placed randomly)
- Monty knows the location of the car
- 2 Goats
- Monty is honest

Pr(car)	$\frac{1}{n}$
Pr(goat)	$\frac{n-1}{n}$
Pr(stay and win)	$\frac{1}{n}$
Pr(switch and win)	$\frac{n-1}{n}$
Winning strategy	Switching is more favorable.

5. *n*-Doors (Part 4.4)

- ***n* Doors**
- Player chooses 1 door and **Monty opens *1* door**
- 1 Car (car placed randomly)
- Monty knows the location of the car
- 2 Goats
- Monty is honest

Pr(car)	$\frac{1}{n}$
Pr(goat)	$\frac{n-1}{n}$
Pr(stay and win)	$\frac{1}{n}$
Pr(switch and win)	$\frac{(n-1)}{(n-2)} \times \frac{1}{n}$
Winning strategy	Switching is more favorable

In general, the probability of winning increases if the player choose to switch his or her initial decision. Also, increasing the number of doors in the game can increase the probability of winning if the player switches decision compared to fewer number of doors.

Part 6: Reflection

Throughout the investigation, I have learnt the importance of assumptions and the effects of decision makings. The experiment from the probability simulation has further inspired me on the close relationship between real life situations and theoretical probabilities of events occurrences. In the process of creating the simulation spreadsheet, I learnt different codings and input functions to be applicable in Geogebra and it has made my overall investigation more comprehensive. Based on the results from the simulation, the experimental value for the 300 trials of Monty Hall problem is really close to the theoretical value calculated using probability. In addition to the Class Monty Hall problem, I have also extended the problem into different possible variations that can further provide me with the opportunity to examine the effect of different factors (i.e behavior of the host, number of doors and probability of car behind door) that may have an influence on the player's final decision.

However, the theoretical probabilities of the different variations of the Monty Hall problem do not guarantee the expected outcome of the actual event. Therefore, simulations for all variations should have been conducted in order to examine actual results of the problem, Additionally, as a further exploration in the future, more variations (such as multiple cars, different prize values or adding more players/hosts into the scenario) can be added. As concluded in this investigation, switching is always more favorable as it increases the probability to win, however is there a case where it does not encourage switching doors? Exploring more outcomes can provide a more comprehensive insight to the problem and simulation of the other variations can be conducted to support the argument.

Part 7: References

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