MATHS COURSEWORK

The SIR model in relation to world epidemics

Name:

IB Candidate Number:

School:

Session:

TABLE OF CONTENTS

Maths Coursework

MATHS COURSEWORK

Understanding the SIR Model in relation to world epidemics

Introduction

My interest in the SIR model was first triggered by an episode called "Vectors" of an American television drama the "Numb3rs." While watching the drama, I was amazed by the way how a simple model could be used to predict a deathly epidemic. Since then, I began researching about models used in epidemiology. Out of other numerous models, the SIR model caught my most attention as it deals with complex situations such as the number of remaining infectives after certain number recovers or dies from the disease. The name is an acronym of three concepts involved in the model: S for Susceptible, I for Infected, and R for the Removed. Through this investigation, I plan to focus on understanding the SIR model and applying it to a real-life epidemic that occurred in the past.

Personal engagement.

Aim.

The SIR Model

Background

First[1] developed by Hermack and McKendrick in 1927, the SIR model is a quantitative model that explains the dynamics of epidemics. This model takes into account the possibility[2] of a disease-contracted patient to recover and become immune to infections in the future.

Variables

Because the SIR mode is used to understand the effect of epidemics over a certain period of time, the independent variable represents time measured in days. For convenience, time will be referred to as t in this investigation.

---

[1] "The SIR Model for the Spread of Disease" by Elise Thorsen
[2] "SIR Model of Epidemics" by Troy Tassier

As for the dependent variables[3], two related sets are considered. The first set of variables regards the number of people that is divided into three categories:

S_t; the number of susceptible individuals who may catch the disease but currently are not infected

I_t; the number of infected individuals who are infected with the disease and are currently contagious

R_t; the number of removed or recovered individuals who cannot get the disease or pass the disease to others

The second set of variables regards the fraction of the total population, N, that also has three categories:

Defines variables.

$$s_t = \frac{S_t}{N};$$ the susceptible fraction of the total population

$$i_t = \frac{I_t}{N};$$ the infected fraction of the total population

$$r_t = \frac{I_t}{N};$$ the recovered fraction of the total population

Both sets of variables present the same information about the epidemic but are used for different equations depending on the situation. For convenience in calculation, the variables representing the fraction of the total population (s_t, i_t, and r_t) will be used in this investigation.

Assumptions

There are several assumptions[4] that lay under the SIR Model.

1) Homogenous mixing

Key terms are explained.

The first is that the model considers homogeneous mixing in which each pair of individuals has equal probability of coming into contact with one another. In other words any uninfected individuals face the same risk of being exposed to the disease through those who had already been infected.

[3] "The SIR Model for the Spread of Disease" by Elise Thorsen
[4] Math0011, "Numbers and Patterns in Nature and Life"

2) Constant population size

   The second assumption is that the population size, N, is large and constant without any net birth, death, immigration, or emigration. Within this assumption[5], the only way for an individual to leave S is to come into contact with a member of I and become a member of I.

3) No latency

   Furthermore, the model assumes that there is no latency, meaning that one gets immediately infectious once infected.

Thus, in order to formulate the SIR model: 1) susceptibles remain disease-free or become infected, 2) infectives pass through an infectious period until they are recovered permanently, and 3) a removed individual is never at risk again.

Mass Action Principle

In the SIR model, the disease is spread when a susceptible comes in contact with an infective and subsequently becomes another infective. Based on the concept of homogenous mixing, the mass action principle shows how the number of encounters between susceptible and infectives is given by the product of S_t and I_t: Not well defined.

$$S_t \bullet I_t$$

Changes in the s, i, r

1) Changes of the susceptible class (s_t)

   It is assumed that the disease is spread once a susceptible individual comes in contact with an infected individual and becomes infected. However, only a proportion α of the encounters between susceptibles and infectives are regarded as infected. Thus, when considering the next time interval, an equation can be derived:

$$s_{t+1} = s_t - \alpha\, s_t i_t$$

[5] "The SIR Model for the Spread of Disease" by Elise Thorsen

2) Changes of the infective class (i_t)

After one time interval, the infective class grows by the addition of the newly infected. During the interval, however, some infectives recover or pass away and are progressed to the removed stage of the disease.

Thus, it is assumed that there is a removal rate γ:

γ = the proportion of the infective class that ceases to be infective and thus moves into the removed class

Thus, an equation over the next time interval can be derived:

$$i_{t+1} = i_t + \alpha s_t i_t - \gamma i_t$$

Deriving equations

In summary, we result with three equations:

$$s_{t+1} = s_t - \alpha s_t i_t$$

$$i_{t+1} = i_t + \alpha s_t i_t - \gamma i_t$$

$$r_{t+1} = r_t + \gamma i_t$$

Using these equations, a set of equations showing the changes in each class can be derived:

$$\Delta s = -\alpha s_t i_t$$

$$\Delta i = \alpha s_t i_t - \gamma i_t$$

$$\Delta r = \gamma i_t$$

This also means that $\Delta s + \Delta i + \Delta r = 0$, which shows a net change in the total population[6] is 0. Also, using the three equations that show what happens in the next time interval, an equation showing the changes during one time interval can be shown.

$$\begin{array}{rl} & s_{t+1} = s_t - \alpha s_t i_t \\ & i_{t+1} = i_t + \alpha s_t i_t - \gamma i_t \\ + & r_{t+1} = r_t + \gamma i_t \\ \hline & s_{t+1} + i_{t+1} + r_{t+1} = s_t + i_t + r_t \end{array}$$

[6] "The SIR Model for the Spread of Disease" by Elise Thorsen

In other words,

$$s_{t+1} + i_{t+1} + r_{t+1} = s_t + i_t + r_t = 1$$

The equation fits for all t as any dead individuals will be counted under r_t.

SIR Model and the Hong Kong Flu

The Hong Kong Flu

One famous epidemic in which the SIR model was applied to was the Hong Kong flu. During[7] 1968 to 1969, an influenza pandemic known as the Hong Kong flu killed an estimate of one million people worldwide. The epidemic was caused by an H3N2 strain of the influenza A virus with origins in the H2N2 antigenic shift, a genetic process in which genes from multiple subtypes reasserted to form a new virus. Growing up in Asia, I did not encounter a big epidemic as the Hong Kong Flu and doubted its existence. The fact that the Hong Kong Flu had a great impact on the Asian population as well as the world triggered me to explore more about the relationship between the Hong Kong Flu and the SIR Model[7].

Personal engagement.

Application of the SIR Model to the Hong Kong Flu in New York City

In order to understand how the SIR Model is applied to real-life epidemics, the population of New York City in the 1960's will be considered in relation to the Hong Kong flu. Hardly anyone in the New York City during the 1960's was immune to this particular virus, thus making almost everyone susceptible:

$$S_0 = 7{,}900{,}000$$

It is also assumed that there was a 10 people infected:

$$I_0 = 10$$

And since nobody was neither removed nor recovered:

$$R_0 = 0$$

If we multiply the initial values by $\frac{1}{N}$ results in equations:

What is N?

[7] Sino Biological Inc., "Hong Kong Flu (1968 Influenza Pandemic)"

$$s(0) = 1$$

$$i(0) = 1.27 \times 10^{-6}$$

$$r(0) = 0$$

Estimated average period of infection was 3 days[8]:

$$\gamma = \frac{1}{3}$$

Infection rate of one new person getting infected every other day:

$$\alpha = \frac{1}{2}$$

Thus, the Hong Kong flu in NYC can be represented by these equations:

$$\Delta s = -\frac{1}{2}\, s(t)\, i(t)$$

$$\Delta i = \frac{1}{2}\, s(t)\, i(t) - \frac{1}{3}\, i(t)$$

$$\Delta r = \frac{1}{3}\, i(t)$$

More explanation for the derivation of Δ and γ would help here.

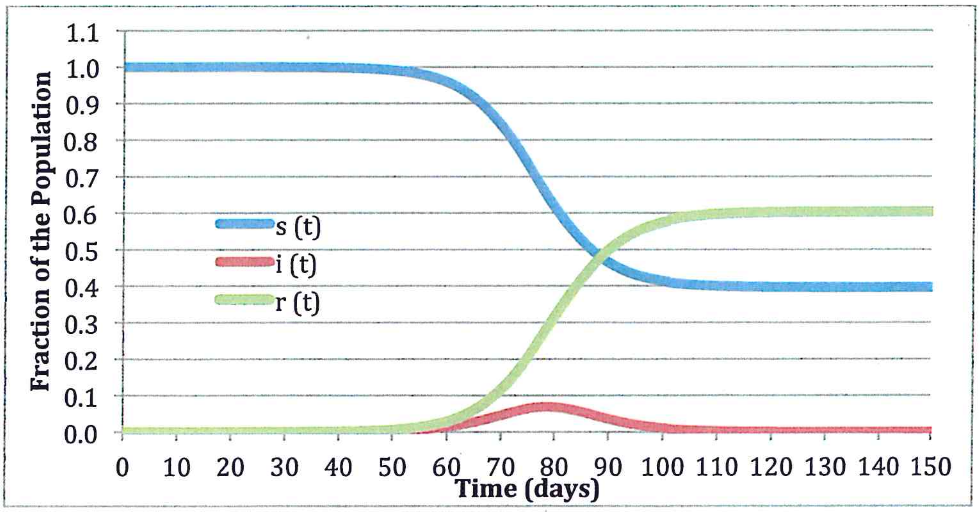
Table 1. Variables that were used

Total	7900000
Initial Infected	10
Delta Time	1
Initial Recovered	0
α	$\frac{1}{2}$
γ	$\frac{1}{3}$

Graphing the SIR Model[9]

Using these equations, a sample graph of the SIR model for the Hong Kong flu in the New York City was graphed. The table on the right indicates the variables used in the graph.

Figure 1. The SIR Model of the Hong Kong flu in New York City



The graph is defined clearly by the data in the appendix. The long table would detract from communication.

[8] JSXGraph Wiki, "Epidemiology: The SIR Model"

[9] The data points used in the graph is indicated in the appendix. The values were calculated in fractions of the populations, which are s_t, i_t, and r_t.

As shown in figure 1, a drastic drop in the class of susceptibles is evidently shown after around 60 days. Although the initial number of susceptibles starts with 7,900,000, the number drops over half by the end of 150 days as s_t reaches 0.5 by day 90. Then, the s_t levels off and stays at 0.4 throughout day 100 to day 150. However, r_t starts to show an increase starting at day 60 and levels off by day 100 at 0.6 and reaches an equilibrium. This shows an inverse relationship between the susceptibles and the recovered as the number of recovered increase as the number of susceptibles decrease. Interestingly, as the number of recovered reaches its maximum point, the rate at which the number of susceptibles decreases reaches a plateau[10]. As for i_t, there is no change until day 60 but shows a positive slope until day 80 at which it reaches its maximum point. It is interesting how although the number of infectives is at its maximum, fewer susceptibles are coming into contact to become infected. After day 80, however, i_t decreases back down and approaches 0 between day 100 and day 150.

Good observation

Solving the SIR Model using Euler's Method

Euler's method for solving a differential equation

After understanding how the SIR Model can be applied to the Hong Kong Flu, I wanted to find a way to solve the equations to grasp a better understanding of what the variables meant in the SIR Model. Out of the various choices, I decided to use Euler's method. The basic idea[11-12] behind Euler's method is to find numerical approximations to solve differential equations. By proceeding in the direction indicated by the direction field, one is making linear approximations from the initial value. For example, Euler's differential equations for y and t are:

$$\frac{dy}{dt} = f\,(y, t)$$

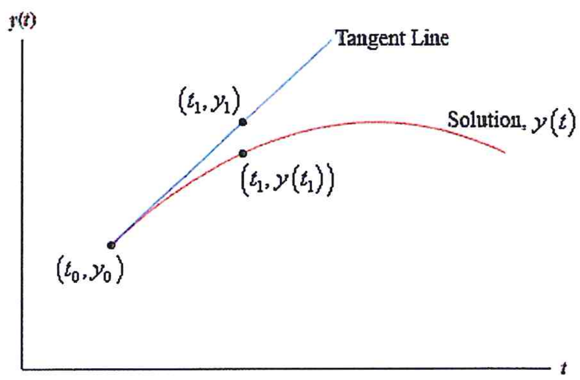
$$t_n = t_{n-1} + \Delta\, t$$

---

10 "The SIR Model for the Spread of Disease" by Elise Thorsen
11 *Single Variable Calculus* by James Stewart
12 "Euler's Method" by Paul Dawkins

$$y_n = y_{n-1} + f\,(y_{n-1}, t_{n-1}) \bullet \Delta t$$

As shown in the graph[13] on the right, Euler's method makes linear approximations of the actual graph using small step values. In this case, the step values are indicated as *t*. Using the Euler's method, it can be said about the SIR model that:



$$s_t = s_{t-1} + \Delta\, s$$
$$i_t = i_{t-1} + \Delta\, i$$
$$r_t = r_{t-1} + \Delta\, r$$

In order to make calculations simpler, the equations that represent the susceptibles, infectives, and the recovered in fraction to the whole population were considered. Because we know the $\Delta\, s$, $\Delta\, i$, and $\Delta\, r$, the equations can be also written as:

$$s_{t+1} = s_t - \alpha\, s_t\, i_t \bullet \Delta\, t$$
$$i_{t+1} = i_t + (\alpha\, s_t\, i_t - \gamma\, i_t) \bullet \Delta\, t$$
$$r_{t+1} = r_t + \gamma\, i_t \bullet \Delta\, t$$

Table 2. SIR Equations for the Hong Kong Flu using Euler's method

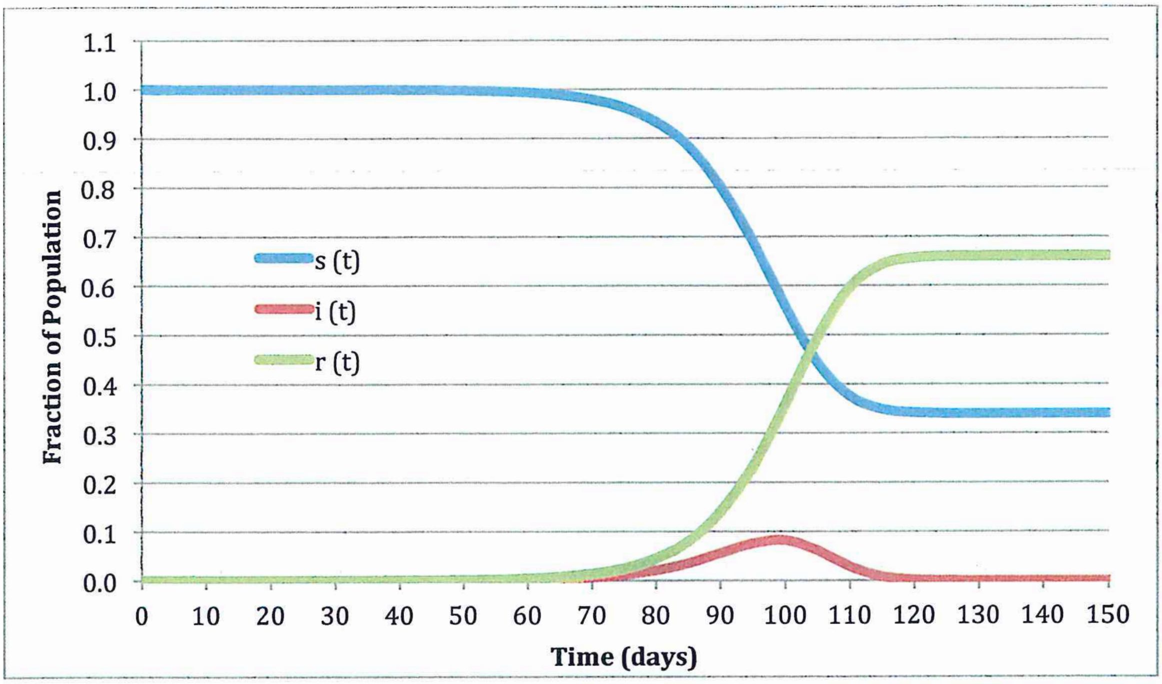
t	s_t	i_t	r_t
t = 0	$s_0 = 1$	$i_0 = 0.00000127$	$r_0 = 0$
t = 1	$s_1 = s_0 - \frac{1}{2} s_0\, i_0 \bullet 1$	$r_1 = i_0 + (\frac{1}{2} s_0\, i_0 - \frac{1}{3} i_0) \bullet 1$	$r_1 = r_0 + \frac{1}{3} i_0 \bullet 1$
…	…	…	…
t = n	$s_n = s_{n-1} - \frac{1}{2} s_{n-1}\, i_{n-1} \bullet \Delta\, n$	$i_n = i_{n-1} + (\frac{1}{2} s_{n-1}\, i_{n-1} - \frac{1}{3} i_{n-1}) \bullet \Delta\, n$	$r_n = r_{n-1} + \frac{1}{3} i_{n-1} \bullet \Delta\, n$

The equations solved with Euler's method are often[14] used with computers in which estimations and repetitions of steps can be done much quickly than humans.

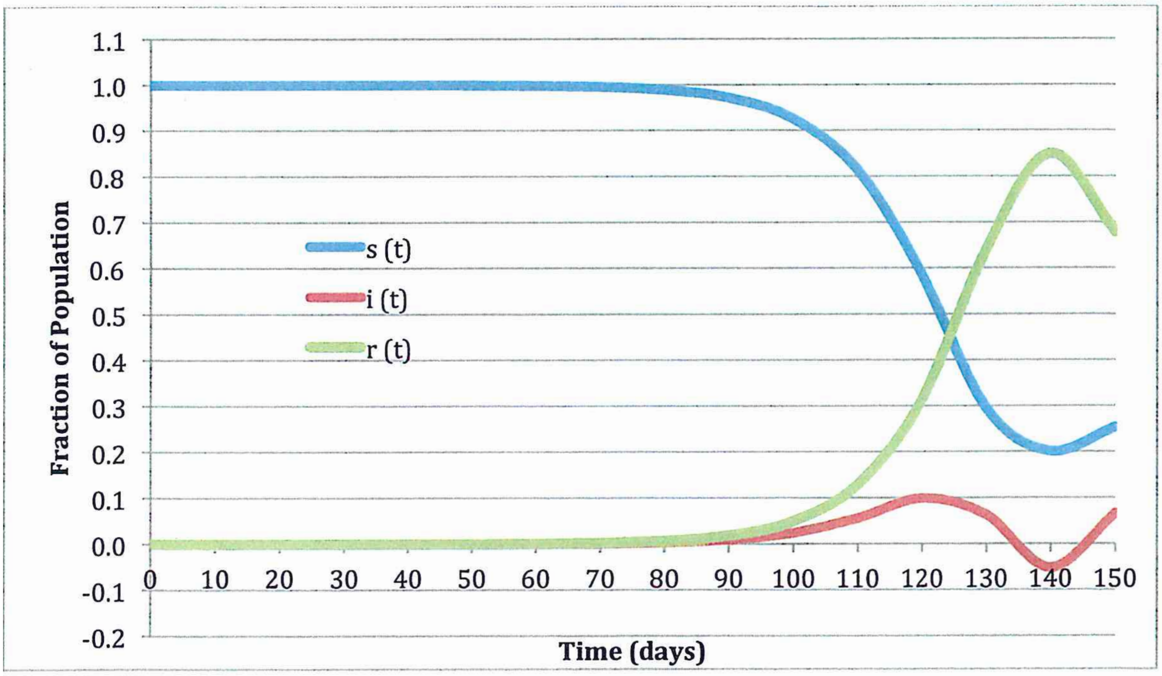
[13] "Euler's Method" by Paul Dawkins

The t-step-value in Euler's method for solving a differential equation

[15]*Figure 3. The SIR Model of the Hong Kong Flu with t = 5*



[16]*Figure 4. The SIR Model of the Hong Kong Flu with t = 10*



---

[14] "The SIR Model for the Spread of Disease" by Elise Thorsen
[15-16] The raw data points used for graphing are in the appendix.

As shown in Figure 3 and Figure 4, depending on the step size of the t-value, the graphing of the SIR model shows different results. In Figure 3, the s_t starts to decrease in day 70 and levels off in day 110. On the other hand, the s_t in figure 4 starts to decrease in day 90 and hit its minimum at day 140 but slightly increases by day 150. The r_t in figure 3 shows an exponential function starting from day 70 and levels off by day 115. In figure 4, however, r_t starts to model an exponential function and hits its maximum at day 140 but slightly decreases by day 150. As for i_t in figure 3, there is relatively no change until day 70 at which the line shows a positive trend and hits its maximum at day 100 but decreases by day 115 and shows no change until day 150. The i_t in figure 4 also shows no change until day 90 at which time the line shows an increasing trend and hits its maximum at day 120. The values, however, decrease back down and actually become negative during day 135 and day 140. A possible reason for such negative values is that the tangent lines extrapolated from the decreasing trend during day 120 to day 135 are too steep that they result in negative values. Because the t-step value is 10, the extrapolations are being made in greater differences. Thus, in figure 3, all three lines starts to show change starting at day 70. As for figure 4, lines start to show change starting at day 90. Such difference could be explained by looking at the difference in the t-step values and where the tangent lines are being made to determine the next point. Thus, in order to have an approximate model of the actual situation, the *t*-step value has to be carefully chosen accordingly to the population size and range of time. Also, it could be said that if the *t*-step-value is too small, the process of finding the s_t, i_t, and r_t values becomes too time-consuming and inefficient since no or very small changes will be observed.

A little too wordy.

Good observation.

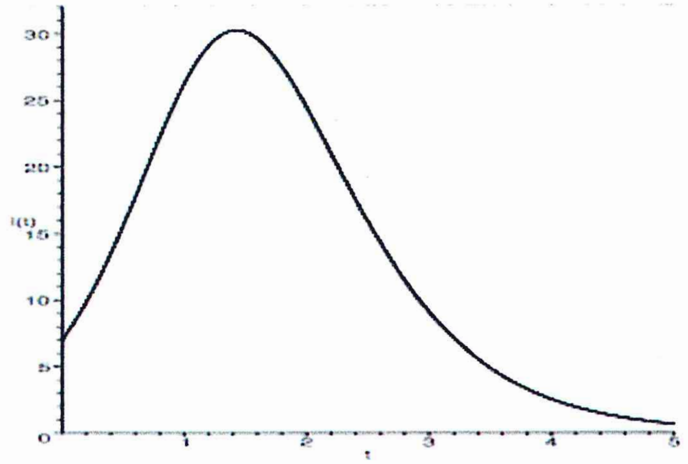
SIR Model and the Eyam Plague

Eyam Plague

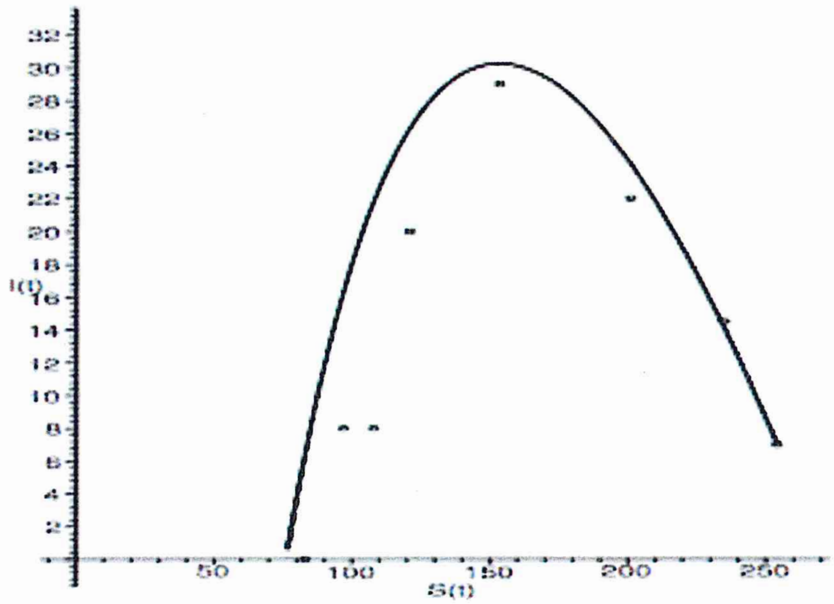
Best known for the plague discovered in August 1666, Eyam is a village in Derbyshire, England. The plague started when a "flea-infested bundle of cloth" was delivered to tailor George Viccars. He died within a week and the plague roamed the village for 14 months and killed at least 260 villagers with 83 survivors[17] out of a population of 350.

Application of the SIR Model to the Eyam Plague

[18]*Figure 5.* ***I*** *as a function of* ***t*** *(in days)*



[19]*Figure 6. Relationship between* S_t *and* I_t



17-19 "Compartmental models for epidemics" by Fred Brauer

Figure 5 outlines the number of infectives in relation to time. It can be seen that the initial number of infectives is 7 at day 0. The number of infectives increases to its maximum of 30 infectives by day 1.5 and decreases until it approaches 0 by day 5. Figure 6 shows the relationship between S_t and I_t. Until S_t reaches 150, I_t increases as S_t increase. This could be interpreted as the time period in which the Eyam Plague continued to spread among the people in the village. When S_t is 150, I_t hits its maximum of 30 but as S_t increases up to 250, I_t begins to drop down and reach 7. The decreasing trend of I_t represents the decreasing number of infectives as less and less people are becoming infected over time. There are limitations to both figures, however, since insufficient data has been found to determine the exact number of susceptibles and infectives. Also, the graphs assume that the transmission[20] of the infection happens directly between people, thus disregarding the various other possibilities of infection.

Applications

The SIR model is used in various fields in today's life, especially in epidemiology and the medical field. One of the most predominant areas of application is vaccine[21]. By estimating the susceptible population, and possible infectives, the range of producing or using vaccination can be planned. Furthermore, by using the SIR model to estimate the dynamics of populations, the best way to cut down on infections can be researched and developed. This is especially effective in terms of money and time as errors are less likely to happen. However, there is always a possibility to get completely different results, as real-life situations are unpredictable and uncertain.

---

[20] "Compartmental models for epidemics" by Fred Brauer
[21] "Modern Applications of the SIR Epidemic Model" by Nik Addleman and Jen Fox

Conclusion

The SIR model is one of many other models that reflects and predicts the various changes in the population. Although the SIR model is widely used by scientists and epidemiologists, it carries a potential limitation[22]. The SIR model assumes that the population is homogeneous and fully mixed. Thus, it is assuming that each individual will have the same number of contacts with every other individual. However, this is not always the case in real-life epidemics as not all susceptibles contact with every other susceptible. Furthermore, in real-life situations, it is harder to track down the number of recovered as deaths can occur from various reasons other than deaths from the actual infection.

Reflection.

Through history classes about the Dark Plague and the epidemics occurring in western parts of Asia, I have always been curious how epidemiology could relate to human catastrophes. As a student interested in the medical field, I have also been intrigued by the various methods used in centers for disease controlling. Through this investigation, I was able to fulfill both of my interests, as I was able to understand a practical model used for world epidemics such as the Hong Kong flu and the Eyam Plague. I also realized how the smallest deviation could entirely affect a population. Beyond epidemiology and medical field, I was also able to explore various connections in the mathematical field such as Euler's method, logistic curves for exponential graphs, and derivatives.

Personal engagement

Overall, the experience helped me explore my self-inquiries and apply math to real-life situations that I had never thought of. For further studies, application of the SIR model to other past epidemics such as the Bombay Plague or the SARS would be helpful.

[22] "Mathematical Approaches to Infectious Disease Prediction and Control" by Nedialko B. Dimitrov and Lauren Ancel Meyers

Appendix 1: Works Cited

Addleman, Nik, and Jen Fox. *Modern Applications of the SIR Epidemic Model*. Los Angeles: Occidental College, n.d. PPT.

Brauer, Fred. *Compartmental Models for Epidemics*. Vancouver: University of British Columbia, 19 June 2008. PDF.

Dawkins, Paul. "Euler's Method." *Paul's Online Math Notes*. Lamar University, n.d. Web. <http://tutorial.math.lamar.edu/Classes/DE/EulersMethod.aspx>

Dimitrov, Nedialko B., and Lauren Ancel Meyers. *Mathematical Approaches to Infectious Disease Prediction and Control*. Austin: The University of Texas at Austin, 2010. PDF.

"Epidemiology: The SIR Model." *JSXGraph Wiki*. JSXGraph Wiki, 20 Feb. 2013. Web. <http://jsxgraph.uni-bayreuth.de/wiki/index.php/Epidemiology%3A_The_SIR_model>.

"Excel Level 2: Susceptible Infected Recovered Model." *Project Succeed*. SHODOR, n.d. Web. <http://www.shodor.org/succeed/curriculum/apprenticeship/Modeling/Excel/LessonPlan2/>

"Hong Kong Flu (1968 Influenza Pandemic)." *Biological Solution Specialist*. Sino Biological Inc., n.d. Web. <http://www.sinobiological.com/1968-Influenza-Pandemic-Hong-Kong-Flu-a-5754.html>

"Population Dynamics I." Lecture. Lecture 2. The University of Hong Kong, Hong Kong. 31 Jan. 2009. *MATH0011 Numbers and Patterns in Nature and Life*. The University of Hong Kong, 31 Jan. 2009. Web. <http://hkumath.hku.hk/course/MATH0011/pdf/lecture02.pdf>

Stewart, James. "Differential Equations." *Single Variable Calculus: Concepts and Contexts*. Belmont: Thomson Brooks/Cole, 2005. 508-09. Print.

Tassier, Troy. *SIR Model of Epidemics*. New York: Fordham University, 15 Sept. 2005. PDF.

Thorsen, Elise. *The SIR Model for the Spread of Disease*. Williamsburg: College of William and Mary, n.d. PDF.

Appendix 2: Raw data for the SIR Model of the Hong Kong Flu

Days	s_t	i_t	r_t
0	1.00000000	0.00000127	0.00000000
1	0.99999937	0.00000149	0.00000042
2	0.99999862	0.00000174	0.00000091
3	0.99999775	0.00000203	0.00000148
4	0.99999674	0.00000238	0.00000215
5	0.99999555	0.00000278	0.00000294
6	0.99999415	0.00000326	0.00000386
7	0.99999252	0.00000381	0.00000493
8	0.99999062	0.00000446	0.00000619
9	0.99998839	0.00000522	0.00000766
10	0.99998578	0.00000610	0.00000938
11	0.99998273	0.00000714	0.00001140
12	0.99997916	0.00000836	0.00001376
13	0.99997498	0.00000978	0.00001651
14	0.99997009	0.00001144	0.00001974
15	0.99996437	0.00001338	0.00002352
16	0.99995768	0.00001566	0.00002793
17	0.99994985	0.00001832	0.00003310
18	0.99994069	0.00002143	0.00003914
19	0.99992998	0.00002508	0.00004622
20	0.99991744	0.00002934	0.00005449
21	0.99990277	0.00003433	0.00006417
22	0.99988561	0.00004016	0.00007550
23	0.99986553	0.00004698	0.00008875
24	0.99984204	0.00005497	0.00010426
25	0.99981456	0.00006431	0.00012240
26	0.99978242	0.00007523	0.00014362
27	0.99974481	0.00008802	0.00016845
28	0.99970081	0.00010297	0.00019749
29	0.99964934	0.00012046	0.00023147
30	0.99958913	0.00014091	0.00027122
31	0.99951871	0.00016484	0.00031772
32	0.99943633	0.00019282	0.00037212
33	0.99933997	0.00022555	0.00043575
34	0.99922727	0.00026382	0.00051018
35	0.99909546	0.00030856	0.00059724
36	0.99894132	0.00036088	0.00069907

Personal example.

37	0.99876107	0.00042204	0.00081816
38	0.99855031	0.00049352	0.00095743
39	0.99830391	0.00057706	0.00112029
40	0.99801587	0.00067468	0.00131073
41	0.99767920	0.00078870	0.00153337
42	0.99728576	0.00092187	0.00179364
43	0.99682608	0.00107733	0.00209786
44	0.99628912	0.00125877	0.00245338
45	0.99566207	0.00147042	0.00286877
46	0.99493005	0.00171721	0.00335401
47	0.99407580	0.00200478	0.00392069
48	0.99307935	0.00233965	0.00458227
49	0.99191762	0.00272930	0.00535435
50	0.99056400	0.00318225	0.00625502
51	0.98898789	0.00370822	0.00730516
52	0.98715420	0.00431820	0.00852888
53	0.98502283	0.00502456	0.00995388
54	0.98254818	0.00584110	0.01161199
55	0.97967860	0.00678312	0.01353955
56	0.97635596	0.00786733	0.01577798
57	0.97251530	0.00911177	0.01837420
58	0.96808463	0.01053556	0.02138109
59	0.96298497	0.01215848	0.02485782
60	0.95713076	0.01400040	0.02887012
61	0.95043065	0.01608037	0.03349025
62	0.94278901	0.01841549	0.03879677
63	0.93410805	0.02101934	0.04487388
64	0.92429089	0.02390012	0.05181026
65	0.91324556	0.02705841	0.05969730
66	0.90089007	0.03048462	0.06862658
67	0.88715842	0.03415634	0.07868650
68	0.87200738	0.03803579	0.08995810
69	0.85542363	0.04206773	0.10250991
70	0.83743077	0.04617824	0.11639226
71	0.81809523	0.05027496	0.13163108
72	0.79753037	0.05424908	0.14822182
73	0.77589773	0.05797953	0.16612401
74	0.75340464	0.06133938	0.18525726
75	0.73029795	0.06420407	0.20549925
76	0.70685390	0.06646077	0.22668659

77	0.68336487	0.06801775	0.24861865
78	0.66012440	0.06881236	0.27106451
79	0.63741204	0.06881664	0.29377259
80	0.61547977	0.06803943	0.31648208
81	0.59454132	0.06652486	0.33893509
82	0.57476543	0.06434755	0.36088829
83	0.55627306	0.06160523	0.38212298
84	0.53913839	0.05841017	0.40245271
85	0.52339281	0.05488039	0.42172806
86	0.50903081	0.05113187	0.43983859
87	0.49601696	0.04727220	0.45671211
88	0.48429306	0.04339628	0.47231194
89	0.47378480	0.03958377	0.48663271
90	0.46440770	0.03589822	0.49969535
91	0.45607200	0.03238751	0.51154176
92	0.44868648	0.02908515	0.52222964
93	0.44216143	0.02601211	0.53182774
94	0.43641065	0.02317889	0.54041173
95	0.43135289	0.02058761	0.54806077
96	0.42691263	0.01823396	0.55485468
97	0.42302048	0.01610891	0.56087188
98	0.41961328	0.01420017	0.56618782
99	0.41663399	0.01249340	0.57087388
100	0.41403140	0.01097317	0.57499670
101	0.41175978	0.00962364	0.57861785
101	0.40977847	0.00842915	0.58179365
102	0.40805143	0.00737457	0.58457527
103	0.40654682	0.00644557	0.58700888
104	0.40523661	0.00562874	0.58913592
105	0.40409613	0.00491174	0.59099340
106	0.40310372	0.00428328	0.59261428
107	0.40224041	0.00373310	0.59402776
108	0.40148961	0.00325198	0.59525968
109	0.40083680	0.00283164	0.59633283
110	0.40026928	0.00246471	0.59726727
111	0.39977601	0.00214463	0.59808063
112	0.39934732	0.00186559	0.59878836
113	0.39897481	0.00162245	0.59940400
114	0.39865115	0.00141070	0.59993941
115	0.39836996	0.00122636	0.60040495

116	0.39812569	0.00106593	0.60080964
117	0.39791350	0.00092636	0.60116140
118	0.39772920	0.00080497	0.60146710
119	0.39756912	0.00069941	0.60173274
120	0.39743008	0.00060764	0.60196355
121	0.39730934	0.00052786	0.60216407
122	0.39720448	0.00045853	0.60233826
123	0.39711341	0.00039828	0.60248958
124	0.39703433	0.00034593	0.60262101
125	0.39696566	0.00030045	0.60273517
126	0.39690602	0.00026093	0.60283432
127	0.39685424	0.00022661	0.60292042
128	0.39680927	0.00019679	0.60299520
129	0.39677023	0.00017089	0.60306015
130	0.39673633	0.00014840	0.60311654
131	0.39670689	0.00012887	0.60316551
132	0.39668133	0.00011190	0.60320804
133	0.39665913	0.00009717	0.60324497
134	0.39663986	0.00008438	0.60327703
135	0.39662313	0.00007326	0.60330488
136	0.39660860	0.00006362	0.60332906
137	0.39659598	0.00005524	0.60335005
138	0.39658503	0.00004796	0.60336828
139	0.39657552	0.00004165	0.60338411
140	0.39656726	0.00003616	0.60339785
141	0.39656009	0.00003140	0.60340978
142	0.39655386	0.00002726	0.60342014
143	0.39654846	0.00002367	0.60342914
144	0.39654377	0.00002055	0.60343695
145	0.39653969	0.00001785	0.60344373
146	0.39653615	0.00001549	0.60344962
147	0.39653308	0.00001345	0.60345474
148	0.39653041	0.00001168	0.60345918
149	0.39652810	0.00001014	0.60346303
150	0.39652609	0.00000881	0.60346638

Appendix 3: Raw data for different step t-values using Euler's Method in relation to the Hong Kong Flu

<T-step-value as 5 days>

Days	s_t	i_t	r_t
0	1.00000000	0.00000127	0.00000000
5	0.99999683	0.00000235	0.00000210
10	0.99999095	0.00000435	0.00000597
15	0.99998008	0.00000804	0.00001314
20	0.99995998	0.00001488	0.00002641
25	0.99992280	0.00002752	0.00005096
30	0.99985401	0.00005090	0.00009636
35	0.99972677	0.00009415	0.00018035
40	0.99949145	0.00017412	0.00033570
45	0.99905637	0.00032190	0.00062300
50	0.99825239	0.00059475	0.00115413
55	0.99676811	0.00109769	0.00213547
60	0.99403276	0.00202186	0.00394665
65	0.98900828	0.00371027	0.00728272
70	0.97983456	0.00676205	0.01340466
75	0.96327034	0.01216889	0.02456204
80	0.93396552	0.02139504	0.04464071
85	0.88400994	0.03604881	0.07994253
90	0.80434118	0.05623703	0.13942306
95	0.69125678	0.07653033	0.23221416
100	0.55900151	0.08251056	0.35848921
105	0.44369269	0.06167695	0.49463163
110	0.37527865	0.02832401	0.59639860
115	0.34870516	0.00816289	0.64313322
120	0.34158906	0.00181023	0.65660199
125	0.34004318	0.00036924	0.65958886
130	0.33972929	0.00007389	0.66019810
135	0.33966653	0.00001473	0.66032001
140	0.33965403	0.00000293	0.66034431
145	0.33965154	0.00000058	0.66034915
150	0.33965104	0.00000012	0.66035011

Personal example.

<T-step-value as 10 days>

Days	s_t	i_t	r_t
0	1.00000000	0.00000127	0.00000000
10	0.99999365	0.00000343	0.00000419
20	0.99997651	0.00000926	0.00001551
30	0.99993022	0.00002500	0.00004606
40	0.99980524	0.00006748	0.00012855
50	0.99946791	0.00018213	0.00035123
60	0.99855773	0.00049127	0.00095227
70	0.99610492	0.00132289	0.00257346
80	0.98951623	0.00354604	0.00693900
90	0.97197192	0.00938842	0.01864092
100	0.92634550	0.02403305	0.04962273
110	0.81503096	0.05603852	0.12893179
120	0.58666532	0.09947705	0.31385890
130	0.29486665	0.06300146	0.64213316
140	0.20198150	-0.05201821	0.85003798
150	0.25451508	0.06710831	0.67837788