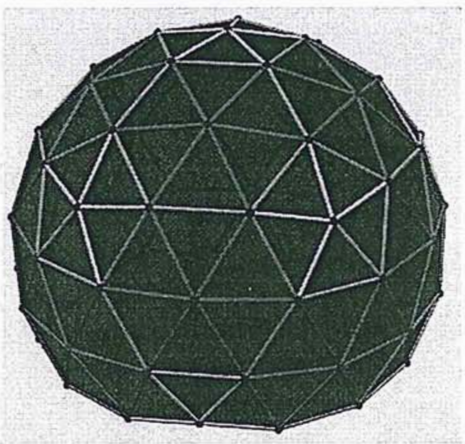
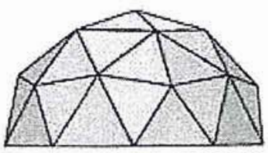
Can a city be covered by a geodesic dome?

I have seen the TV series called “Under the dome”, which was turned from a book of Stephen KING with the same name. The topic is that a dome suddenly encloses a whole town of which source is unknown to the people. Besides being able to stand with its gigantic size, the dome is very durable and resistant to every force. It is for sure that it is impossible to create a dome with this resistance, however, what I wondered was if it is possible to enclose an area as big as a city with a dome. This is why I decided to do this project.

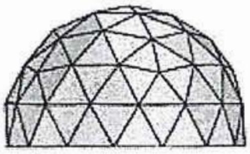
In this project, I want to investigate what must the volume and the height of the dome in order to be able to enclose an area of a town. I will take the surface area of a town as 500km^2, as it is an average town size and the town in the TV series is not a large one. I will investigate the minimum and maximum volumes of the dome to enclose this much of surface area. For this investigation, geodesic domes seemed to be best options for particular reasons. Very first reason is that it has abase very close to the circular base, which has the ability of covering the most area with the same perimeter compared to other two-dimensional shapes. Additionally, geodesic domes are constructed by triangles, which is the most durable shape and has the ability to share the pressure on one side to the other two sides. This ability gives it not only to stand but also to carry weights that are put on them. I will construct demonstration in which the dome is constructed by metal frames and triangles are filled with glass. Thickness of the glass will be determined on how much weight the dome can carry.



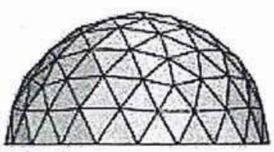
Geodesic domes have different types according to their frequency. These types are called 2V, 3V, 4V and it goes on. The number represents the frequency of the geodesic dome. Simply, it indicates the number of connections between the centers of two pentagons. As this number increases, number of different lengths and the number of triangles, pentagons and hexagons are also increasing. For a certain surface area, like 500km^2, as the number of triangles decrease, their size must increase. This means that the size of triangles are larger in 2V than in 3V for a specific surface area. My hypothesis is that as the triangles increase in numbers and decrease in size, they will be more durable to the stress of the structure.



2V Dome



3V Dome



4V Dome

General Calculation:

First of all, in order to be able to calculate the number of triangles used in each dome type, perimeter of the surface of the dome must be calculated. This can be done by relating the formula for the area of a circular base with the formula of the perimeter of the circular base. Base is not completely circular but very intimate to circular shape.

Town area was decided as $500km^2$. Using the simple formula, radius of this area can be found as:

$$Area\ of\ circle = \Pi r^2 = 500km^2$$

$$r^2 = \frac{500km^2}{\Pi}$$

$$r = \sqrt{\frac{500km^2}{\Pi}}$$

$$r = 12{,}62km$$

From the knowledge of the radius, perimeter of the structure can be calculated by:

$$Perimeter = 2\Pi r$$

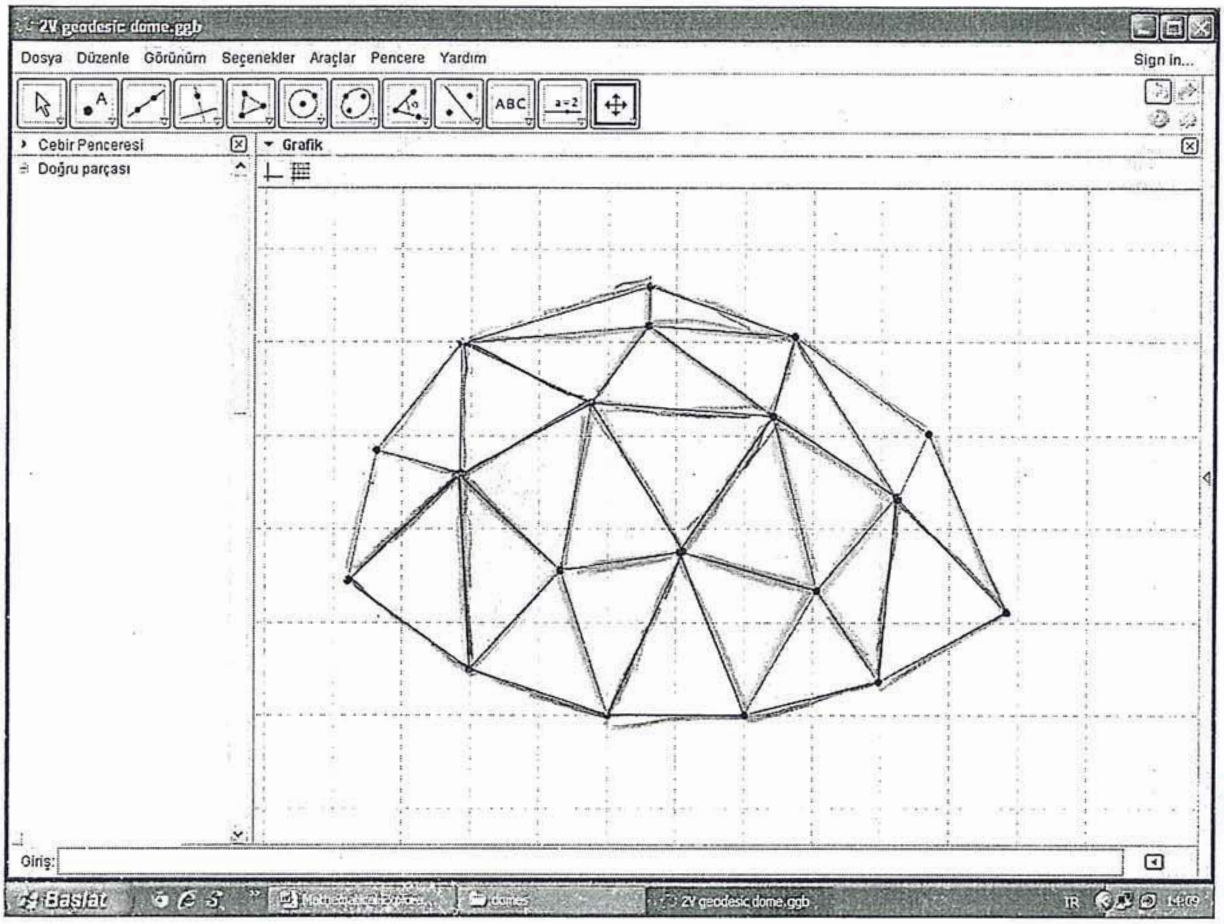
$$Perimeter = 2 \bullet \Pi \bullet 12{,}62km$$

$$Perimeter = 79{,}29km$$

The perimeter of the structure will be 79,29km means that when the number of triangles are found for the type of the dome, length of the base of each triangle can be calculated by simple dividing. This will help to find the area of each triangle.

Physics suggests that as the matter decreases in size, it becomes more and more durable. The reason behind this is that the ratio of its surface area to its volume increases. In this exploration, I will show that as the number of connections between the centers of pentagons increases, areas of the triangles will decrease and their amount will increase, which will cause the triangles to be more durable so the overall structure.

In this type of geodesic dome, two different lengths will be used for the triangles.



Here is my graphing of 2V dome from the side view. Blue colored lines are equal in length and red colored lines are equal in length with each other. Blue colored line can be represented as B and red colored lines can be represented as R. It must be also dedicated that |B|>|R|. However, in order to be able to present a three-dimensional shape I have changed the lengths of some lines since GeoGebra does not allow three-dimensional construction.

This drawing is half side of the whole shape. In total, the shape has five pentagons that have sides on the base and one pentagon at the top. There are ten equal lengths of lines at the base and they are all blue colored, so B.

$$Perimeter = 79{,}29km = 10|B|$$

$$|B| = 7{,}92km$$

Since this is a pentagon, length of R can also be calculated easily by using cosine theorem. The angle between two R is 72 degrees and this degree faces with a B, so:

$$|B|^2 = |R|^2 + |R|^2 - 2 \bullet |R| \bullet |R| \bullet \cos(72°)$$

$$7{,}92km^2 = 2 \bullet |R|^2 - 2 \bullet |R|^2 \bullet \cos(72°)$$

$$62{,}73\,km^2 = [2 - 2\cos(72°)] \bullet |R|^2$$

$$62{,}73km^2 = 1{,}38 \bullet |R|^2$$

$$|R|^2 = 45{,}45km^2$$

$$|R| = 6{,}74km$$

As indicated before, length of |B| is slightly more than length of |R|. There are two different types of triangles. First type is equilateral triangle that have sides of length B and second type is isosceles triangle that has two sides of length R and one side of length B. Their areas can be calculated using the sine theorem:

$$Area(isosceles) = \frac{|R| \bullet |R| \bullet \sin 72}{2} \qquad Area(equilateral) = \frac{|B| \bullet |B| \bullet \sin 60}{2}$$

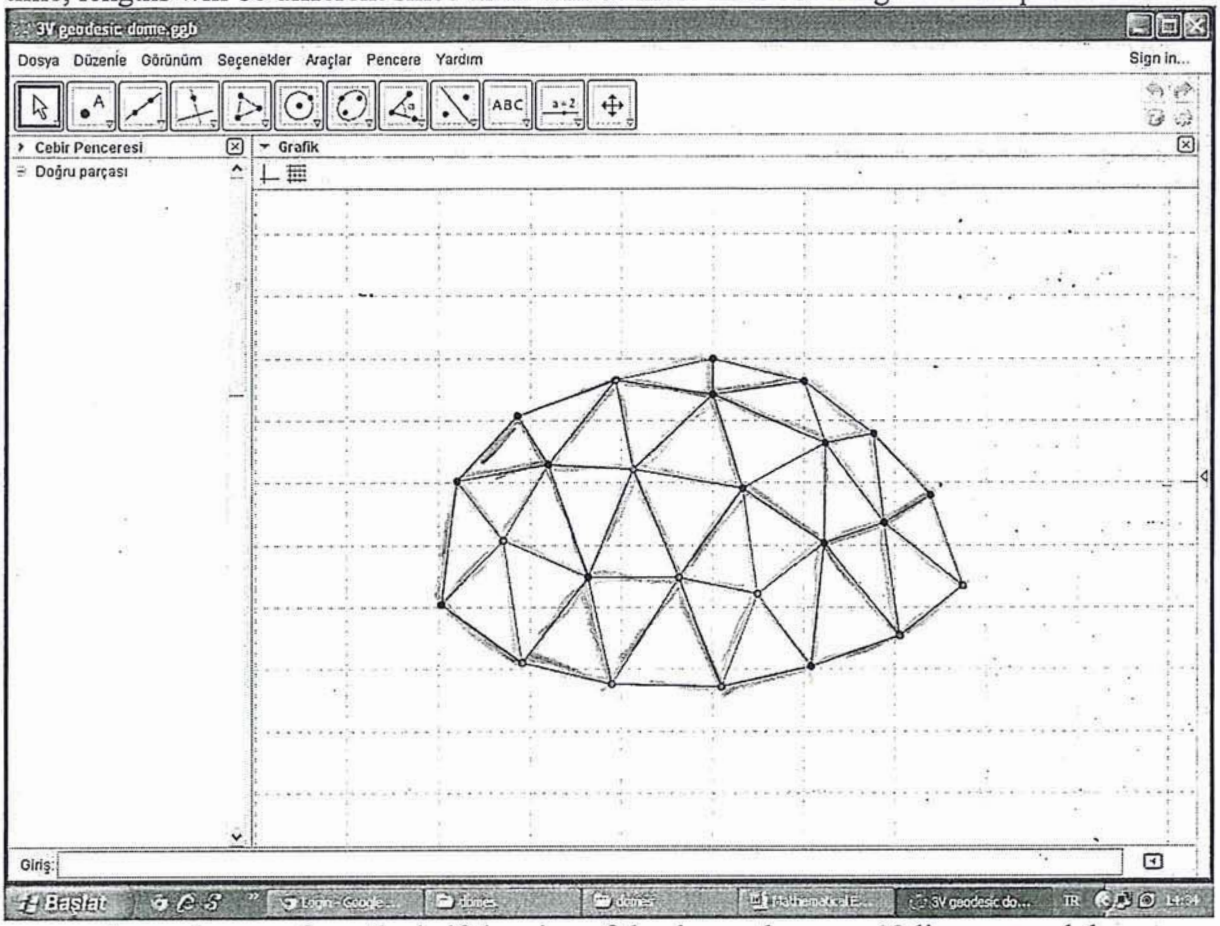
$$Area(isosceles) = \frac{6{,}74km \bullet 6{,}74km \bullet 0{,}95}{2} \qquad Area(equilateral) = \frac{7{,}92 \bullet 7{,}92 \bullet 0{,}87}{2}$$

$$Area(isosceles) = 21{,}59km^2 \qquad Area(equilateral) = 27{,}28km^2$$

These values are extremely high for construction. Not only it is impossible to melt down and fill an area of 21,59km^2 of glass, it is also obvious that a glass of this size will be very fragile, unable to stand against its weight. This can result to the conclusion that 2V geodesic dome does not accommodate to the dome in the TV series.

3V Dome

Same processes that are applied on 2V Dome calculations will be repeated, except this time, lengths will be different since there will be more lines covering the same perimeter.



As can be seen from the half drawing of the dome, there are 12 lines around the perimeter this time.

$$Perimeter = 79{,}29km = 12|B|$$

$$|B| = 6.61$$

$$|B|^2 = |R|^2 + |R|^2 - 2 \bullet |R| \bullet |R| \bullet \cos(72°)$$

$$6.61^2 = |R|^2 + |R|^2 - 2 \bullet |R| \bullet |R| \bullet \cos(72°)$$

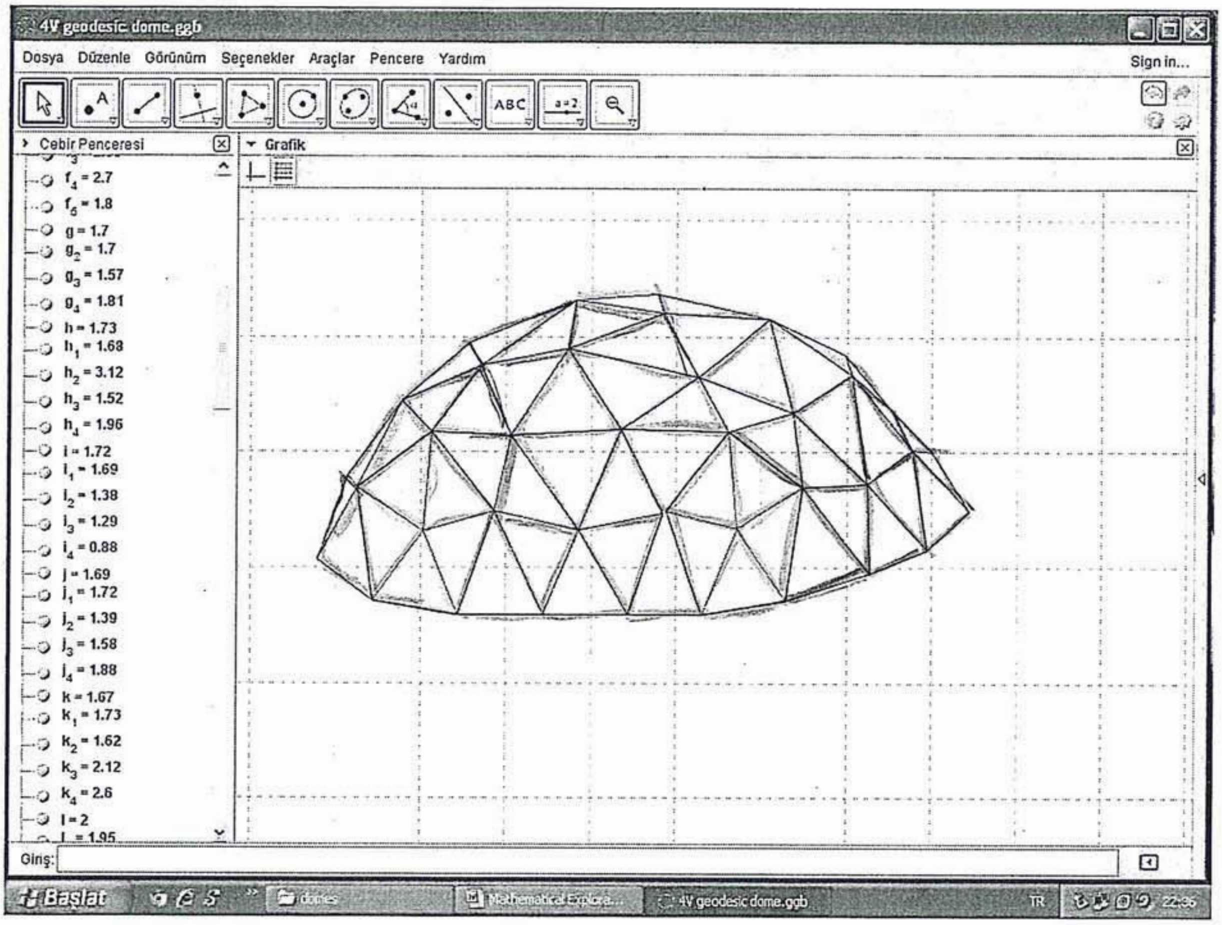
$$43{,}69 = 1{,}38|R|^2 \qquad |R|^2 = 31{,}66 \qquad |R| = 5{,}62$$

This already shows that there are observable changes in the lengths. Since finding the area is a multiplication process, this change is expected to be even more observable.

$Area(isosceles) = 15{,}04km^2$ $Area(equilateral) = 19{,}01km^2$

The change in the type showed its effect observably in the areas of the triangles even in this level. However, this effect of change will be greater as the number of connecting increases. By other words, this change is expected to increase exponentially.

4V Dome:



Exactly same calculations will be carried out for this dome and 5v dome in order to have enough data to produce a graph which would help to make guesses about other types of domes.

$Perimeter = 79{,}29km = 18|B| \Rightarrow |B| = 4{,}41km$

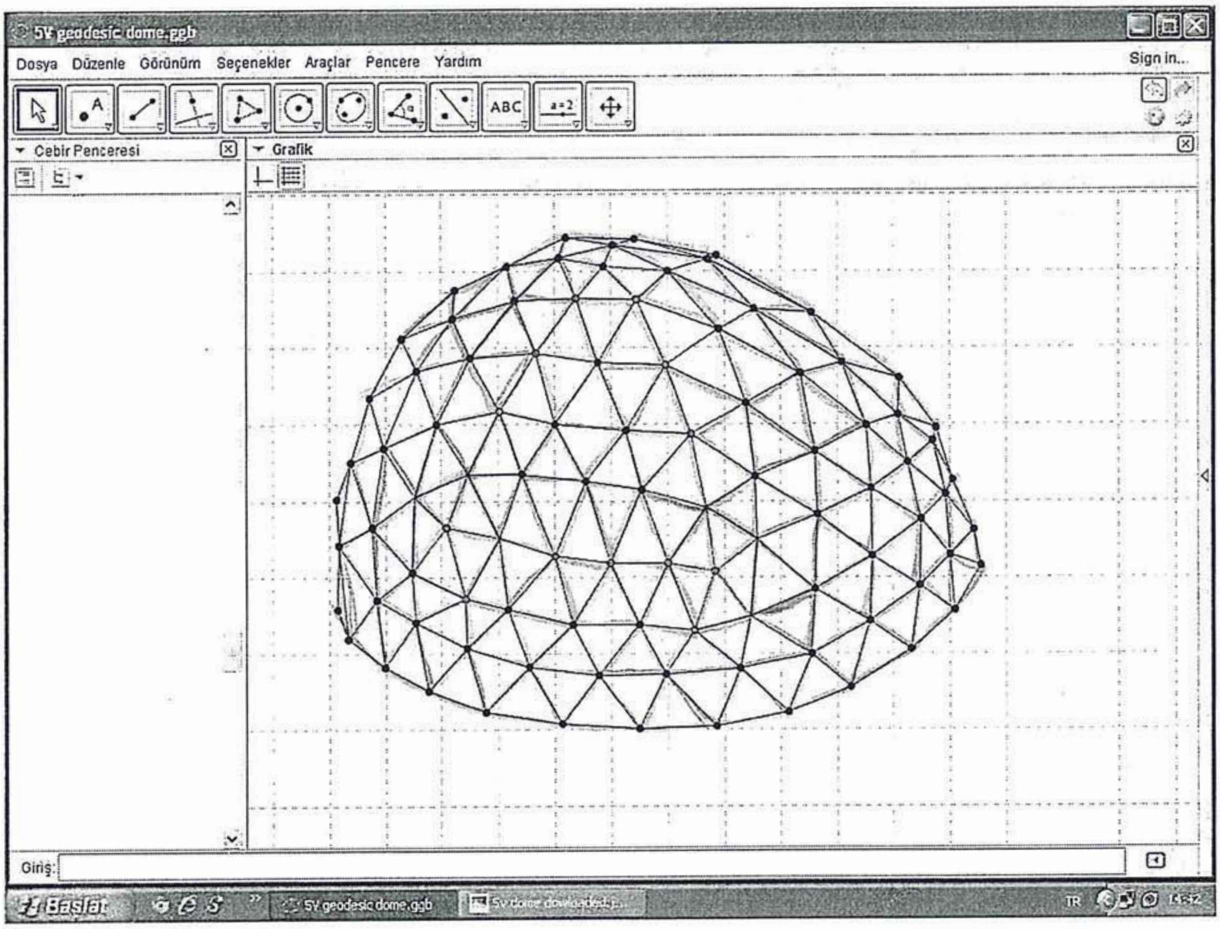
$|B|^2 = |R|^2 + |R|^2 - 2 \bullet |R| \bullet |R| \bullet \cos(72°)$

$|B|^2 = 1{,}38|R|^2 \Rightarrow |R|^2 = 14{,}09$

$|R| = 3{,}75km$

Areas are significantly reduced compared to 2V and 3V domes. However, it must also be mentioned that decrease from 2V to 3V is less than the decrease from 3V to 4V. This already shows that it has an exponential pattern.

5V Dome



After drawing the shape for 5V geodesic dome and finding the number of lines covering the same perimeter, the same calculations can be carried out for the last time to obtain the last area data of the types.

$$Perimeter = 79{,}29km = 24|B| \Rightarrow |B| = 3{,}30km$$

$$|B|^2 = 1{,}38|R|^2 \Rightarrow |R|^2 = 7{,}91 \qquad |R| = 2{,}81km$$

$$Area(isosceles) = \frac{2{,}81km \bullet 2{,}81km \bullet 0{,}95}{2} \qquad Area(equilateral) = \frac{3{,}30 \bullet 3{,}30 \bullet 0{,}87}{2}$$

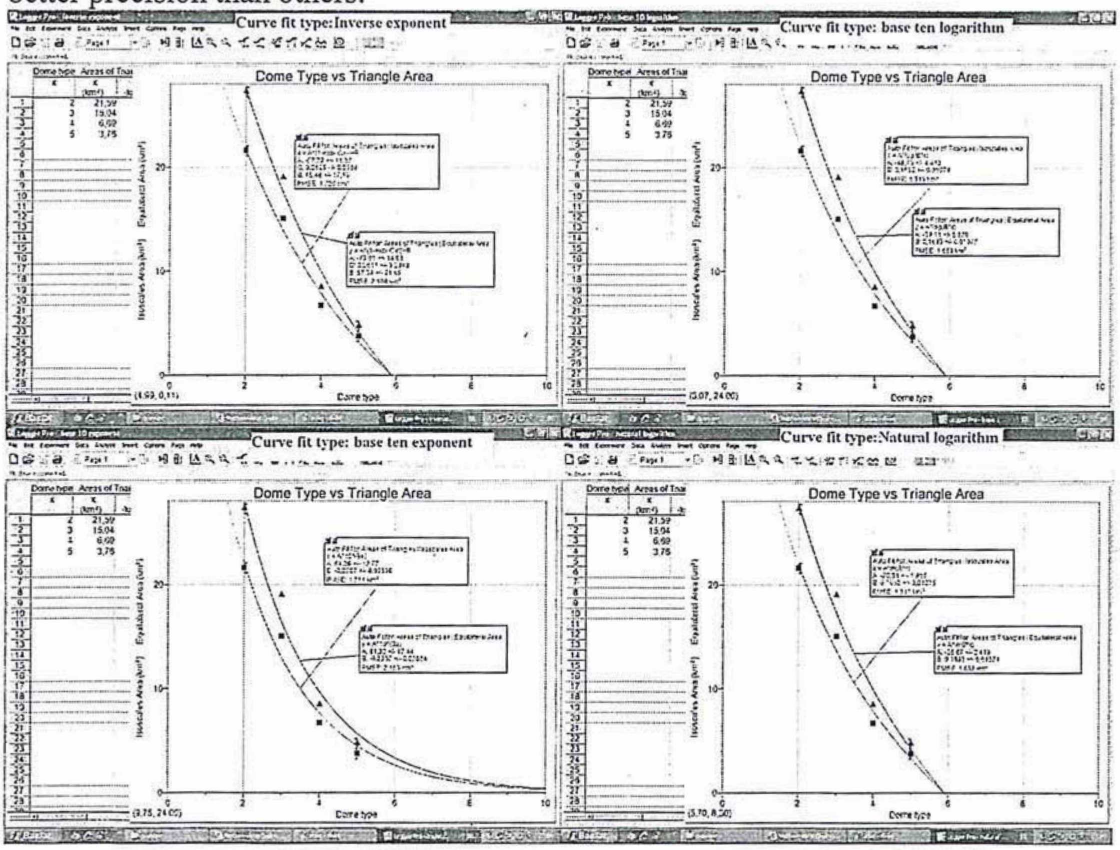
$$Area(isosceles) = 3{,}76km^2 \qquad Area(equilateral) = 4{,}74km^2$$

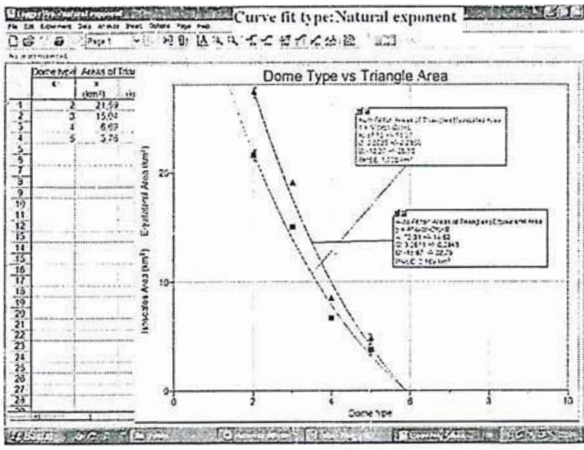
So far, it was measured and calculated that how the lengths and the areas of two different lines and triangles have been changed by the change of the dome type. As dome type changed, number of connections between the centers of pentagons changed so by also looking at the calculations, it can be concluded that lines became shorter in length and triangles have decreased in area.

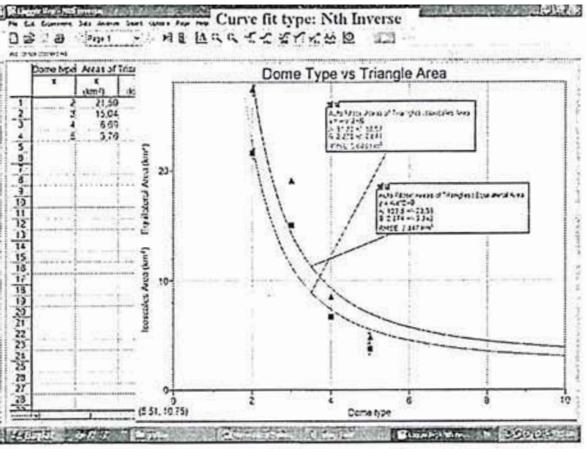
Additionally, by constructing a table and a graph, an estimation can be obtained using the data values calculated. By other words, by calculating the change in lengths and areas, values for lengths and areas can be estimated from a graph.

Type of dome	Length of B (km)	Length of R (km)	Area of isosceles triangle (km^2)	Area of equilateral triangle (km^2)
2V	7,92	6,74	21,59	27,28
3V	6,61	5,62	15,04	19,01
4V	4,41	3,75	6,69	8,46
5V	3,30	2,81	3,76	4,74

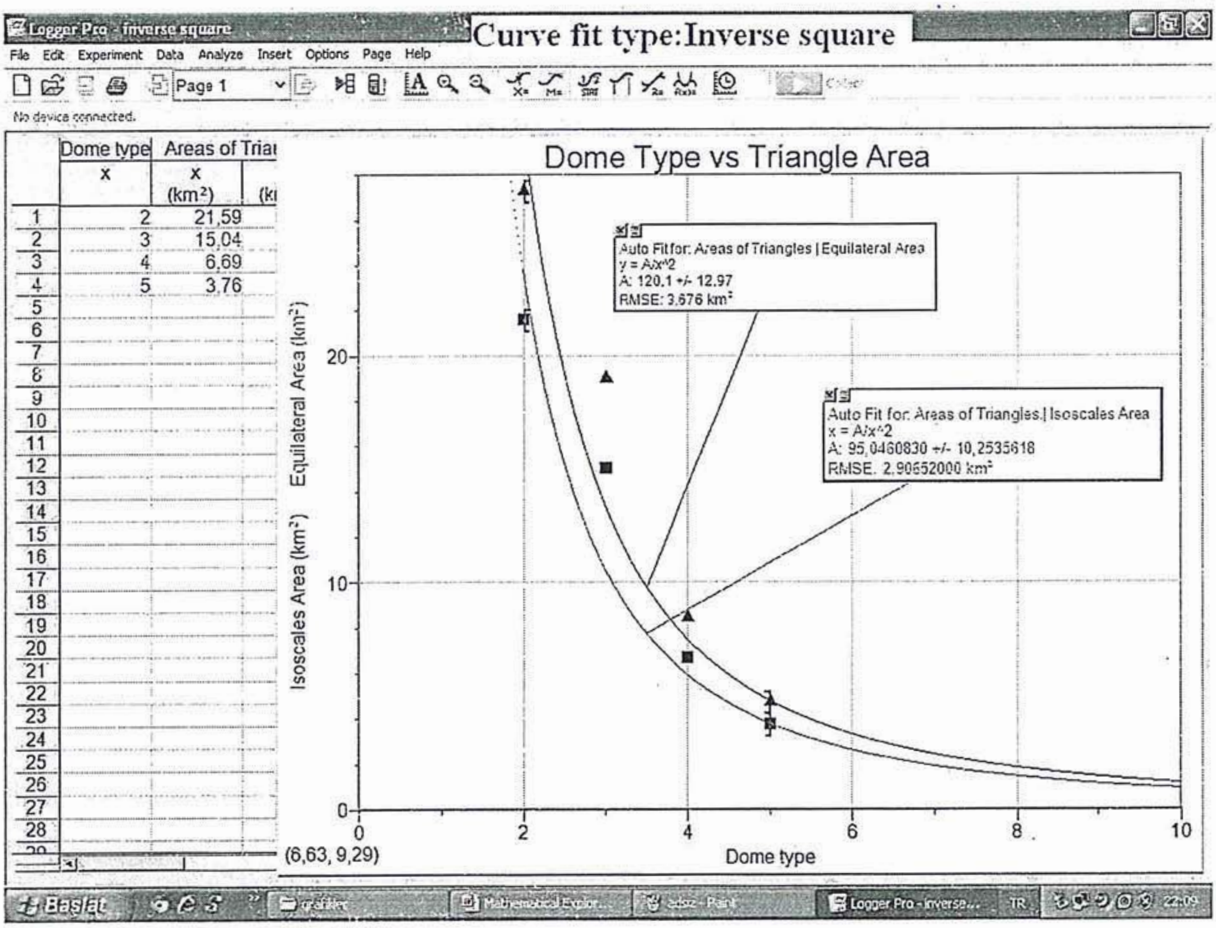
A graph of this table can be used to estimate former data for other types of geodesic dome. There were several types of functions for graphing these data. Some of them showed a better precision than others:







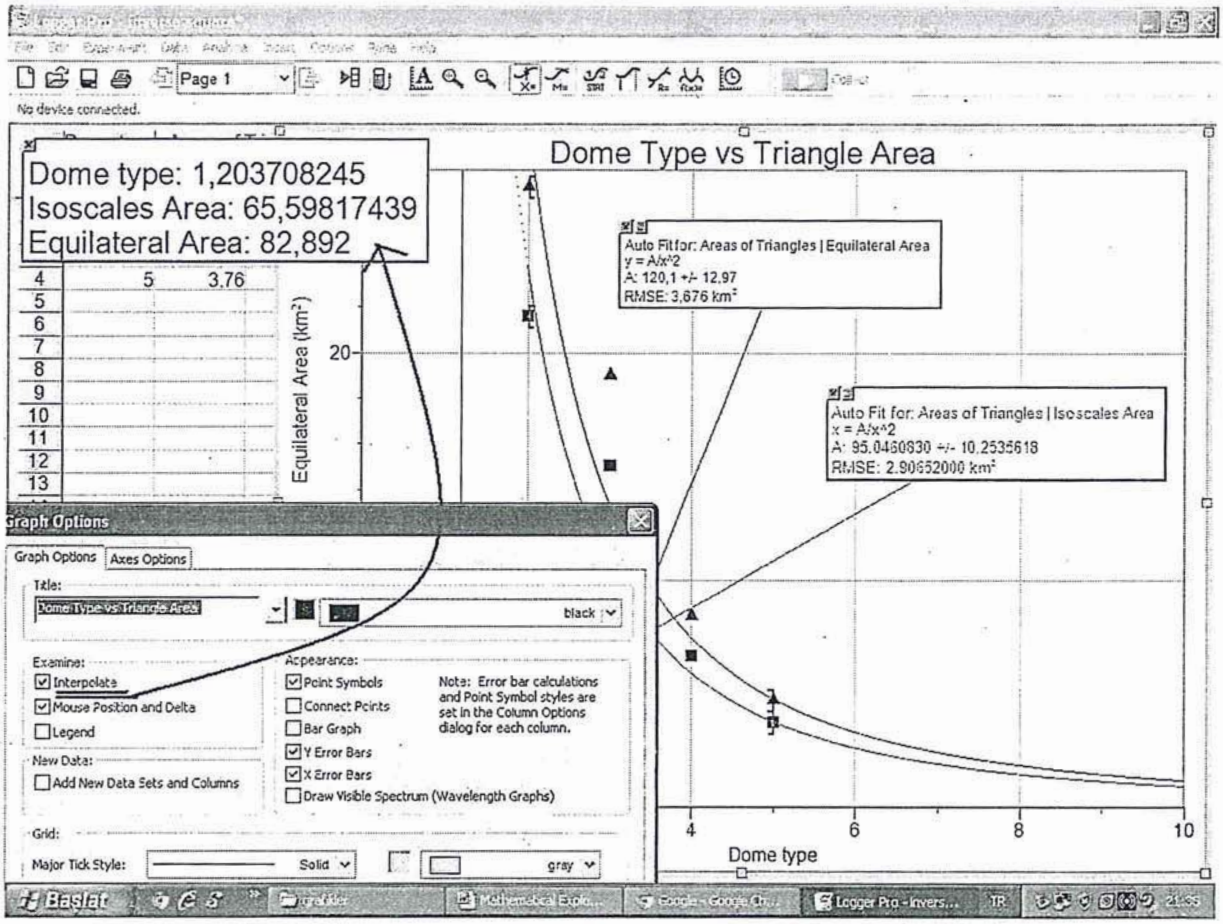
They are the graphs which are mostly precise to the data I have collected by my calculations. However, some of them are not suitable for this investigation. These are: natural logarithm, base ten logarithm, natural exponent and inverse exponent graphs. The reason that they are not suitable is that their range includes the value of y=0, which means that the areas of the two types of the triangles are 0. This means that there are no triangles after that value so no dome at all. The function that seems to be fitting best is the inverse square function.



It can be seen that as the dome type increases, so the connections between the centers of the pentagons, areas of the triangles also are decreasing. This decrease shows a curve trend and it can be observed that through the points x=6, 8, and 10, decrease is less than compared to lower values like x=2, 3 etc.

the durability will also not change. This increase of dome type after this point will only cause to increase of the triangles so higher construction value.

The change in increase can be calculated by finding the function type and inserting the dome values to the function to calculate the areas of the triangles. However, an easier way to do this is using the graph and reading the values in the graph that corresponds to integer numbers greater than 6. For this, the box that indicates the addition of interpolation must be chosen so that it opens up a new plane which reads the values on the graph in the position where the cursor is set.



The reason that the values readed must be integers is that they symbolize the number of the connective sticks between centers and bases of equilateral triangles so they can not be decimal. The reason they are greater and equal to 6 that decrease of areas can be observed to decrease after x=6.

Dome type: 6,005150001
Isoscales Area: 2,635642981
Equilateral Area: 3,3305

Dome type: 7,009860862
Isoscales Area: 1,934262917
Equilateral Area: 2,4442

Dome type: 8,001629565
Isoscales Area: 1,484490483
Equilateral Area: 1,8759

Dome type: 9,000399399
Isoscales Area: 1,173304503
Equilateral Area: 1,4826

Dome type: 10,010200983
Isoscales Area: 0,9485248358
Equilateral Area: 1,1986

Dome type: 11,003016544
Isoscales Area: 0,7850743158
Equilateral Area: 0,99205

Now, by simple subtraction, the change in decrease can be found. The new data can be formed to a table.

Dome Type	Area of Isosceles Triangle (km^2)	Area of Equilateral Triangle (km^2)	Change in decrease as y_1-y_2 where y_1=x and y_2=x-1	
			Δ Area Isosceles (km^2)	Δ Area Equilateral (km^2)
6	2,64	3,33	-	-
7	1,93	2,41	0,71	0,92
8	1,48	1,88	0,45	0,53
9	1,17	1,48	0,31	0,40
10	0,95	1,19	0,22	0,29
11	0,79	0,99	0,16	0,20

A graph for these values can also be drawn where it would be unnecessary. It can be already observed that there is a very small difference in change of decrease when going from x=10 to x=11 compared to when going from x=6 to x=7. This means that after x=10, the change in decrease of the areas of the triangles are really small. When there is no observable decrease in the areas, durability of the whole structure will not be affected by this decrease. So in result, the best decision to use a dome type to cover up a town would be making the dome 10V dome. This type will be as durable as any larger types of its own kind and will have a very less construction cost compared to higher-valued types.

I have chosen dome type values as close as possible to be integers. It is not possible to specify a x value in the graph to measure the y value so the cursor must be adjusted on the value.

Conclusion

In this exploration, my aim was to investigate the type of the dome that would be best fitting to cover up an area of a town. This idea came to my mind by watching a TV series called "Under The Dome". For the calculations, I have taken the town area as 500km^2, which is the average area of a town, including the fact that the town, which is taken under a dome, is not already a big one. From this estimation of area, which is accepted to be circular since dome has a circular base and circle is the shape which has the highest perimeter with any given area, I calculated the radius of the basement. From this radius, perimeter of the base of the dome was calculated.

After these steps, I have drawn the shapes of domes by obeying the simple formula that there must be x number of connective lines between the centers of any two pentagons in a XV dome. I have constructed these domes by using GeoGebra software. However, I simplified the domes in drawings. I have taken only two types of lines in the construction but there are more than two types in geodesic domes (except from 2V dome). The reasons that I simplified the domes are that firstly there is a limit of pages and mathematical use for this exploration and secondly it can be seen that lines that are classified to be in different lengths are very close to each other, which can be found by a geodesic dome calculator[1].

After drawing the domes and calculating the areas of the triangles, I have graphed these areas and observed that areas are decreasing as the dome type increases. Since the areas

[1] Check the web site for calculators: http://www.domerama.com/calculators/

are decreasing, durability of the triangles are increasing so the durability of the whole structure does. This means that the durability increases as the dome type increases.

However, after I chose the best curve fit for the data, I have realized that areas of the triangles start to decrease less after a point. This meant that after a level, increasing the dome type would not effect the durability of the structure in observable amounts. I wanted to prove this by reading the area values of the triangles in specific dome types. After reading these values and calculating the area decrease compared to the last dome type, I have recorded the data to a table and observed that after 10V dome, areas of the triangles do not change in huge amounts. This meant that after 10V dome, durability of the structure would not change but the money spent for the construction would increase since there would be more connective metals in each increase of the dome type.

To sum up, the best dome type in order to cover a town was chosen to be 10V dome since it is as durable and less consuming compared to higher types of its own kind.

SOURCES

-http://www.earth360.com/math_geodesic_dome_education.html

-http://sci-toys.com/scitoys/scitoys/mathematics/dome/dome.html

-GeoGebra Software

-LoggerPro Software

-http://www.domerama.com/calculators/